Risk Neutral Valuation of With-Profits Life Insurance

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Executive Summary

- **Develop Framework in which the following embedded options in participating life-insurance contracts can be evaluated separately**
  - **Interest Rate Guarantee**: Consider surplus distribution mechanism (management rules)
  - **Surrender Option**: Develop valuation model in order to price complex, path-dependent options

- **Practical Implementation**:  
  - Prerequisites and Downsides of the risk-neutral approach

- **Key Results**:  
  - Guarantees are currently offered below their risk neutral value
  - Financial strength of the insurance company considerably affects value of a contract
Introduction

- **Guarantees in Participating Life Insurance Contracts:**
  - **Year-to-Year Cliquet Style (Ratcheting) Guarantee:** Liabilities (including already credited surplus) earn a guaranteed rate of interest p.a. plus some surplus (if possible)

    → Value of the guarantee depends on *regulatory prerequisites* (guaranteed rate of interest, participation levels) and on the company’s *surplus distribution mechanism* (management decision)

  - **Deferred annuities: Guaranteed annuity factors** (not considered here)

- **Current Problems:**
  - Low interest rates
  - plunging stock markets
  - Insurers are stuck with old, non-hedged guarantees
Introduction (2)

- **Literature:**
  - Mostly focusing on unit-linked products
  - Mostly valuation of contract as a whole – no consideration of the embedded options separately (Implementation?)
  - Most models are not adequate for some markets (e.g. the German market)
  - Mostly, the financial strength of the issuer is not taken into account (e.g. via reserve quota)

- **Target of this paper:** Fill the gap –
  Valuate and Analyze offered guarantees / options of a participating contract in a framework taking into account the prevailing surplus distribution rules (in e.g. the German market) and the financial strength of the issuer
Model

- Insurer’s financial situation

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_t$</td>
<td>$L_t$</td>
</tr>
<tr>
<td>$A_t$</td>
<td>$A_t$</td>
</tr>
</tbody>
</table>

- Development of the assets and liabilities (general):

- Market value of assets
- Book value of Policyholder’s account
- Reserves (Valuation Reserves, equity, etc.)

Annual return of the “reference portfolio” $A$

- “-”: before dividend payment
- “+”: after dividend payment

- Reserve Quota $R_{t-1}/L_{t-1}$

\[
A_t^- = A_{t-1}^+ (1 + r_t) \\
A_t^+ = A_t^- - D_t \\
L_t = L_t \left( A_t^-, A_{t-1}^+, x_{t-1}, L_{t-1}, \ldots \right) \\
D_t = D_t \left( A_t^-, A_{t-1}^+, x_{t-1}, L_{t-1}, \ldots \right)
\]
Model

- **Considered distribution schemes:**
  1. **MUST-case:**
     - Only obligatory payments considered
     - Year-to-year cliquet style guarantee on the liabilities (minimum rate $g$)
     - Proportion $\delta$ of earnings on book value have to be credited to policyholder’s accounts
     - “Rest” of the earnings on book values paid out as dividend
     - “Rest” of the earnings on market values remains in the company (Reserves)
     - e.g. $BV(\cdot)$ denotes the function transforming earnings on market values to earnings on book values (depends on regulatory framework and management decisions) – for details see paper

\[
L_{t+1} = (1 + g)L_t + [\delta \ BV(A_{t+1}^- - A_t^+ ) - gL_t]^+ 
\]

→ **Obligatory payments should be considered in any meaningful valuation – in particular interesting for regulatory organs**
Model(2)

- Considered distribution schemes:
  - 2. IS-case:
    - In addition to obligatory payments, typical surplus distribution schemes of German Insurers are considered (model by Kling/Richter/Russ)
    - Target interest rate \( z \) is credited to the policyholders’ account as long as the reserve quota stays within a given range \([a, b]\)
    - If crediting target rate leads to a quota below \( a \) or above \( b \), the company credits exactly the rate that leads to \( x_t = a \) or \( x_t = b \), respectively (however, the guarantees must still be fulfilled!)
    - Dividends amount to a portion \( \alpha \) of any surplus credited to the policy reserves

→ Such corporate political issues might be of interest for companies’ actuaries as well as for customers, who are interested in the value of their product.
Risk Neutral Valuation

- **General Approach:**
  - **As usual:** There exists a risk neutral measure \( Q \) and a numéraire process \((B_t)_{t\in[0,T]}\)
  - **General pricing formula:** \( P^* = E_Q[B_T^{-1}L_T] \)

- **Problems:**
  1. **Underlying security not traded:** Company’s portfolio is subject to management and investment decisions
  2. **We want to price the embedded option rather than the whole contract:** A possible hedging strategy could not be implemented within the company, since then the underlying would change

- **Solution:**
  1. Approximate reference portfolio by traded benchmark portfolio
  2. Alternative Approach: Price the cash flows
Risk Neutral Valuation(2) - Cash Flow Approach

- **Relevant Cash Flows:**
  - **Dividends** are paid out and reduce the value of the reference portfolio (but not the asset allocation)
  
  Value:
  \[
  Z_0 = E_Q \left[ \sum_{t=1}^{T} B_t^{-1} D_t \right]
  \]

  - If the return of the reference portfolio is so poor, that granting the **minimum interest guarantee** would result in negative reserves, capital is needed in order to fulfill obligations (capital shots \( C_t \)) - they increase the value of the reference portfolio, asset allocation stays the same
  
  Value:
  \[
  F_0 = E_Q \left[ \sum_{t=1}^{T} B_t^{-1} C_t \right]
  \]

  - **Similarly:** Value \( WAO_0 \) of the **Surrender Option** as the supremum of the values of all possible surrenders
Risk Neutral Valuation(3) - Cash Flow Approach

Equilibrium Condition for a fair contract:

\[ F_0 + WAO_0 + R_0 = Z_0 + E_Q[B_{T}^{-1}R_T] \]

\[ \leftrightarrow P^* = E_Q[B_{T}^{-1}L_T] = L_0 \]

since

\[ P^* = E_Q[B_{T}^{-1}L_T] = L_0 + R_0 + F_0 - Z_0 - E_Q[B_{T}^{-1}R_T] + WAO_0 \]
Numerical Analysis

- **Model market** by a geometric Brownian motion \((A)\) with constant volatility and the numéraire \((B)\) by riskless asset with constant interest rate \(r\)

- **Simulation methods:**
  1. **Monte Carlo Simulation:**
     → No consideration of the Surrender Option, but allows for pricing value of Dividends, Interest Guarantee and Reserve-Delta separately

  2. **Discretization Approach (Extension of Lukkarinen and Tanskanen’s (2004) Approach):** Use Black-Scholes PDE in order to solve valuation problem within one period and use arbitrage arguments at the transitions
     → Consideration of the Surrender Option possible
### Selected Results - Parameters for the Simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guaranteed rate of interest $g$</td>
<td>3.5%</td>
</tr>
<tr>
<td>Minimum participation rate $\delta$</td>
<td>90%</td>
</tr>
<tr>
<td>Insurer’s initial reserve quota $x_0$</td>
<td>10%</td>
</tr>
<tr>
<td>Target interest rate $z$</td>
<td>5%</td>
</tr>
<tr>
<td>Reserve corridor $[a,b]$</td>
<td>[5%,30%]</td>
</tr>
<tr>
<td>Portion provided to share holders $\alpha$</td>
<td>5%</td>
</tr>
<tr>
<td>Asset volatility $\sigma$</td>
<td>7.5%</td>
</tr>
<tr>
<td>Risk free rate of interest $r$</td>
<td>4%</td>
</tr>
<tr>
<td>Time horizon $T$</td>
<td>10 years</td>
</tr>
<tr>
<td>Initial investment $L_0$</td>
<td>10,000</td>
</tr>
</tbody>
</table>
## Selected Results(2) - Values

<table>
<thead>
<tr>
<th></th>
<th>MUST-Case</th>
<th>IS-Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium</td>
<td>10,000.00</td>
<td>10,000.00</td>
</tr>
<tr>
<td>+ Interest Guarantee</td>
<td>+868.42</td>
<td>+998.99</td>
</tr>
<tr>
<td>- Dividends</td>
<td>-238.16</td>
<td>-74.36</td>
</tr>
<tr>
<td>- Reserve-Delta</td>
<td>-275.76</td>
<td>-20.30</td>
</tr>
<tr>
<td>Value of a “European Contract”</td>
<td>10,354.50</td>
<td>10,904.33</td>
</tr>
<tr>
<td>+ Surrender Options</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Value of an “American Contract”</td>
<td>10,354.50</td>
<td>10,904.33</td>
</tr>
</tbody>
</table>
Selected results(3) - Influence of the guaranteed rate $g$

- Obviously, the value of a contract increases with increasing guaranteed rate.

- However, the level is very high: In the Must-Case, the value of a contract exceeds the insured’s investment at a level of 2.75% (current level) and in the Is-Case even with a 0% guarantee.
Selected results(4) - Influence of the initial reserve quota $x_0$

- **Is-Case:** The value of a contract increases with increasing reserve quota, since more is paid out if the reserves are high.

- **Must-Case:** Value of a contract also increasing in $x_0$. 

The graph illustrates the relationship between the value of a contract and the initial reserve quota $x_0$. The graph shows two lines: the solid line represents the **Must-Case** and the dashed line represents the **Is-Case**. As $x_0$ increases from 0% to 40%, the value of the contract increases for both cases, with the **Must-Case** showing a steeper increase than the **Is-Case**.
Selected Results(4) - Interaction of guaranteed rate $g$ and target rate $z$

- In order to treat contracts alike (same risk-neutral value), **contracts with a lower guaranteed rate should be granted a higher target rate**, if the reserve situation is good!

- **However**: Relationship depends on other parameters, too: In company with higher reserves, the “slope” is smaller.
Outlook

- **Extensions of our model:**
  - Stochastic interest rate model
  - More advanced model for the underlying assets
  - **Empirical Research:** Is it possible to find an adequate traded benchmark portfolio?
  - **Implementation:** How to hedge the options/guarantees?

THANK YOU!
Backup (1) – Possible Question to Tanskanen/Lukkarinen’s (2004) approach

- Within one period: Black-Scholes Problem → Heat Equation
- \( t=1,2,3,\ldots,T \) – No Arbitrage condition:

\[
\begin{align*}
V(v-, A_{v-1}, A_v, L_{v-1}, x_{v-1}) &= \\
V(v, A_v, A_v, L(A_{v-1}, A_v, L_{v-1}, x_{v-1}), X(A_{v-1}, A_v, L_{v-1}, x_{v-1}))
\end{align*}
\]

- Numerical Implementation: Mesh... know values at \( t+1 \) and solve for values at \( t \)