THREE FUNDAMENTAL ISSUES OF INSURANCE

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7. September 2005
1. Certum ex Incertis

The coat of arms of the Institute of Actuaries

reflects the situation of life insurance at founding date 1848

Great Britain 1844 - 1853:

- 335 new insurance companies planned
- 148 new insurance companies founded
  
  only

- 59 of these survived the ten year period
On this historical background

main task of actuary:

make insurance work
keep promises given to the insured
avoid ruin

Question: what makes insurance work?

Obviously general preconditions

political
economical
societal

}\normal conditions
What makes insurance work under normal conditions?

1. the collective

2. the reserve

And, in addition, I like to reflect on the role of

3. Profitability

⇒ THREE FUNDAMENTAL ISSUES
2. The Collective

Object of insurance: risk contract

life insurance policy
(individual, group)

motor insurance contract
(individual, fleet)

reinsurance contract
(treaty, facultative acceptance)

Main Idea:

Individual risks belong to
Collective (set of individual risks sharing fluctuation)
What is the role of the collective?

Half-correct answer:

Diversification ⇐ too superficial!

Good way to think:

Insurance Claim ⇔ Random Experiment
Example (Bichsel 1964, Motor Liability Insurance)

Individual Driver ⇔ $D(\lambda)$

- dice can show number of car accidents in one year
  \[ k = 0, 1, 2, \ldots \]

- each driver has a different dice $D(\lambda)$

- Insurance company does not know which dice the insured has
  \[ \lambda \text{ unknown} \]

Poisson distribution

\[ \mathbb{P}(D(\lambda) = k) = e^{-\lambda} \frac{\lambda^k}{k!} \]

Which frequency should the company use for the premium calculation?
Collective

all Swiss car drivers

draw at random

Individual risk

\[ D(\lambda) \]

\[ u(\lambda) = \frac{c^\gamma}{\Gamma(\gamma)} \lambda^{\gamma-1} e^{-c\lambda} \geq 0 \]

\[ \gamma = 1 \]

\[ c = 6.4 \]

\[ E[\lambda] = \frac{\gamma}{c} = \frac{1}{6.4} \]

\[ \lambda_0 \]
Risk of Insurance Company

\[ F \sim \text{frequency used in calculation} \]
\[ \lambda \sim \text{correct frequency (unknown)} \]
\[ k \sim \text{number of claims actually occurring} \]

\[
\mathbb{E}[(k - F)^2] = \mathbb{E}[(k - \lambda)^2] + \mathbb{E}[(\lambda - \lambda_0)^2] + (\lambda_0 - F)^2
\]

Individual Variance
\[
\sigma^2 = \frac{\gamma}{c}
\]

Collective Variance
\[
\tau^2 = \frac{\gamma}{c^2}
\]

(Bias)\(^2\)
\[
\left(\frac{\gamma}{c} - F\right)^2
\]

risk components

A

B

C
Diversification \( k \leftarrow \bar{k} \)

\[ i) \quad n \text{ years} \quad \begin{array}{c}
\text{A} \\
\rightarrow \\
\frac{A}{n}
\end{array} \]

\[ ii) \quad n \text{ years} \quad \begin{array}{c}
\text{A} \\
\rightarrow \\
\frac{A}{nN}
\end{array} \]

\[ \text{B} \\
\rightarrow \\
\frac{B}{N} \]

- No Diversification of  \( \text{C} \)

- Unless \( F \) is adjusted to the experience there will be antiselection \( \Rightarrow \) increase of  \( \text{C} \)

Experience Rating \( \Rightarrow \) Credibility
Tariff Revisions
General Case

- frequency no longer constant in time
  
  \( \lambda \)  regression

  \( \lambda _1 \)  stochastic curve

- from frequency to
  
  \( \lambda \)  severity

  \( \lambda _1 \)  risk premium
The collective

- Present in day to day insurance reasoning but in vague intuitive form

- Mathematical Modelling must clarify and quantify the intuitive background

- Collective is heterogeneous

- Same or similar concept could bring new dimension of thinking in wider area of finance
3. The Reserve

Financial Trading ⇒ Prices
No Arbitrage

Insurance ⇒ Reserves

Illustration

Life insurance policy ⇒ stream of stochastic payments

\[ X = (X_0, X_1, \ldots, X_n) \]

\[ \text{time } \begin{cases} 0 & \sim \text{begin of policy} \\ n & \sim \text{end of policy} \end{cases} \]

\[ X_k = \text{Benefits paid in } (k-1, k] - \text{Premium received at time } k \]
Define the payment stream after time $k$

$$X^{(k)} = (0, 0, \ldots, X_{k+1}, \ldots, X_n)$$

$$R[X^{(k)}/\mathcal{F}_k] \sim \text{reserve for } X^{(k)} \text{ at time } k \text{ given the information } \mathcal{F}_k$$
Let us try to play a game with the policy (cash stream \(X\))

**Company (A):** keeps contract until the end

**Company (B):**

i) plans to get out of the contract at time \(k\), if \(F_k \in \mathcal{F}_k\) occurs e.g. index linked policy
   index exceeds given level

ii) has to transfer \(R[X^{(k)}/\mathcal{F}_k]\) to the new risk carrier

\[
\begin{array}{cccccccc}
\text{(A)} & X_0 & X_1 & \ldots & X_k & \ldots & X_{n-1} & X_n \\
\text{(B)} & \text{if } F_k \text{ occurs} & X_0 & X_1 & \ldots & X_k + R[X^{(k)}/\mathcal{F}_k] & \ldots & 0 & 0 \\
& \text{if } F^c_k \text{ occurs} & X_0 & X_1 & \ldots & X_k & \ldots & X_{n-1} & X_n \\
\end{array}
\]

**No Arbitrage** \(Q[A] = Q[B]\) \(\Rightarrow\) Derive Reserve
We need a method to assign a

Value $Q$ to a payment stream $X$

$$X \quad \mapsto \quad Q[X]$$

$\uparrow$

functional

$Q$ is a valuation method

i) traditional actuarial

$$Q[X] = E \left[ \sum_{k=0}^{n} v^k X_k \right]$$

$$v = \frac{1}{1 + i}$$

ii) market consistent valuation

replace $v^k$ by $\varphi_k \sim$ deflator

choose deflator to reflect ”market prices”

⇒ program for actuaries in market driven economy
The reserve

- Ideas expressed can be carried over to Non-Life Insurance

- No arbitrage in a situation where there is no market for life insurance policies becomes a fairness condition to protect the policy holder

- For risk prices e.g. in mortality one may get an idea from the YRT-rates charged by the reinsurer
4. A Further Step In Reserving

We have defined

\[ R[X^{(k)}/\mathcal{F}_k] \sim \text{reserve at time } k \text{ given the information } \mathcal{F}_k \]

**Understanding:**

\[ R[X^{(k)}/\mathcal{F}_k] \div R_k \text{ (abbreviation) stands for a money amount e.g. in Euro} \]

**New Understanding:** (Multidimensional Valuation)

\( R_k \) is the value of a **portfolio** of financial instruments

\( (VaPo)_k \)
To understand this better, think of an endowment policy (non participating)

face amount 1
\[
\begin{align*}
  n &= 5 \\
  x &= 50
\end{align*}
\]

In the classical actuarial model the valuation at \textbf{beginning of the policy} (for \(l_{50}\) persons)

payments at time (age)
\[
\begin{align*}
  50 & \quad l_{50} \times (-P) \\
  51 & \quad l_{51} \times (-P) \quad d_{50} \times 1 \\
  52 & \quad l_{52} \times (-P) \quad d_{51} \times 1 \\
  53 & \quad l_{53} \times (-P) \quad d_{52} \times 1 \\
  54 & \quad l_{54} \times (-P) \quad d_{53} \times 1 \\
  55 & \quad d_{54} \times 1 \quad l_{55} \times 1
\end{align*}
\]
This cash stream is represented by the **VaPo** (for \(l_{50}\) policies)

<table>
<thead>
<tr>
<th>instruments</th>
<th>number of units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z^{(50)})</td>
<td>(- Pl_{50})</td>
</tr>
<tr>
<td>(Z^{(51)})</td>
<td>(- Pl_{51} + d_{50})</td>
</tr>
<tr>
<td>(Z^{(52)})</td>
<td>(- Pl_{52} + d_{51})</td>
</tr>
<tr>
<td>(Z^{(53)})</td>
<td>(- Pl_{53} + d_{52})</td>
</tr>
<tr>
<td>(Z^{(54)})</td>
<td>(- Pl_{54} + d_{53})</td>
</tr>
<tr>
<td>(Z^{(55)})</td>
<td>(+ l_{55} + d_{54} = l_{54})</td>
</tr>
</tbody>
</table>

↑

*Zero Coupon Bonds*

If you sell this policy, you **sell the above VaPo**

No question about existence!!

**The VaPo can be constructed for all type of policies**  \(\Rightarrow\) more general financial instruments

**Non-Life contracts**
Important property of VaPo

\[(VaPo)_{k+1} = (VaPo)_k + \text{cash stream from policy}\]
+ premiums
- benefits
(- reinsurance premium)

VaPo **refinancing** is **automatic** from the cash stream of the policy

**Challenge of today**

- Rethink Actuarial Valuation
- Construction of VaPo
5. Profitability

We have treated

\[
\begin{align*}
\text{premium calculation} & \implies \text{collective} \\
\text{liabilities} & \implies \text{reserve Valuation Portfolio}
\end{align*}
\]

What about the **assets** of the insurance company?

(a) assets used to cover the liabilities of insurance contracts

(b) assets for other liabilities

(c) free assets

**I talk only on** (a) assuming that from (b) and (c) no debts can be imposed on (a)
Suppose at time $k$

\[ S_k \] is the \textbf{portfolio} of assets for (a) (fund)

\[(\text{VaPo})_k\] is the \textbf{portfolio} of the \textbf{aggregated} (Valuation Portfolio) insurance liabilities

\textbf{Important:} Portfolio not Value of portfolio

- Assets and liabilities speak the same language
- Assets and liabilities are comparable
Solvency

Insurance Company is solvent if (cash flow from policies added to $S$)

\[
\begin{align*}
S_k & \text{ can be exchanged for } (\text{VaPo})_k \\
S_{k+1} & \text{ can be exchanged for } (\text{VaPo})_{k+1} \\
\vdots & \vdots
\end{align*}
\]

Observe:

i) $S$ may be continuously restructured (but in a selffinancing manner)

ii) ”exchange” means mostly by market mechanism
Solvency Preserving Investment Strategy

Funds covering the Valuation Portfolio

(a) must be of sufficient value to start with (the condition of the accountant)

(b) must transport this value into future years to cover maturing liabilities (the condition of the actuary)

Profitability hence becomes a constrained optimization problem

$S$ can only be optimized for profit under the side conditions (a) and (b)
The Rational Solution for Solvency Preservation

Value of $S_k$: $Z_k$ \{ market value \\
Value of VaPo$_k$: $V_k$

We control: $X_k = \frac{Z_k}{V_k}$ degree of coverage (Deckungsgrad)

The side condition (a) and (b) are fulfilled if

$$X_k \geq 1 \text{ at start } (k = 0)$$

$$\geq 1 \text{ through the whole period of insurance coverage}$$
$$ (k = 1, 2, \ldots, n)$$
The side conditions (a) and (b) can be expressed as a financial contract

**Option:** You may, at all time points, exchange $S$ for the VaPo

equivalently $X$ for 1

This option is called **Margrabe-Option**
(Gerber-Shiu, Transactions SOA)

Protected portfolio $M + S$

garantees that side conditions are fulfilled
Numerical Example: \( X \sim \text{geometric Brownian motion} \ \sigma = 5\% \)

\( k = 0, 1, 2, \ldots, 12 \)

\( Z_0 = V_0 \Rightarrow X_0 = 1 \quad \text{Price}(M_0) = 2\% \)

We hedge \( M + S \) against VaPo
Remarks:

- You will probably not find an institution that sells this option for you.
- The price for $M$ is spent at strike time.
- Conventional way of attacking the problem.

   capital at risk
Excursion into Decisional Behaviour

Decision in an insurance company

Result of a rather complex decision process

Theoretical Concept

- The rational solution of the problem is chosen by the decision maker

More realistic description

- Different interests and aspects are superposed to the rational analysis and lead to several proposals

- The chosen solution results from social behaviour
In this complex situation

finding an optimum is quite difficult

solving a constrained optimization problem is almost hopeless

Optimization of the profitability for an insurance company is such a constrained problem
Let me return to the title

**THREE FUNDAMENTAL ISSUES OF INSURANCE**

Why not

**THREE FUNDAMENTAL ISSUES FOR THE ACTUARY?**

In 1848 it was the actuaries who made insurance work
They were the experts of risk management

In 2005 Actuaries have no longer the monopoly for risk management in insurance
Several professions compete for excellence in this field

Who is going to cope with these fundamental issues in the future?

**SEIZE YOUR CHANCE!**