Risk-Based Solvency Capital Requirements

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Overview

• Solvency II
• Available and Required Capital
• Diversification
• Industry Practice
• Equilibrium approach to value and measure capital and risk transfers
Solvency II

- New EU-system to assess overall solvency based on prospective risk-oriented approach (initiated 2001)
- 3 pillars: quantitative requirements, supervisory activities, public disclosure
- Pillar 1: Solvency Capital Requirement (SCR) and Minimum Capital Requirement (MCR), valuation of assets and liabilities, group *diversification*, etc.
- Committee of European Insurance and Occupational Pension Supervisors (CEIOPS) in charge to advice the Solvency II project through three specific calls for advice (Aug 2004-Mar 2006)

http://europa.eu.int/comm/internal_market/insurance/solvency_en.htm
Solvency II: organisation/compatibility

IASB → EC
CEA → IC
Groupe Consultatif → CEIOPS
IAIS → EC
IAA → IC
Basel II → CEIOPS
EU States → Solvency II
Switzerland → Solvency II
US, Canada, Australia → Solvency II

(Lamfalussy)
Available Risk Capital (C)

\[ C = A - L \]

available capital  value of assets  value of liabilities

• Depends on choice of and valuation principles for assets and liabilities

• Market consistent valuation of assets:
  • marked to market if available
  • marked to model else (e.g. risk-neutral valuation)

• Value of Liabilities:

<table>
<thead>
<tr>
<th>Statutory reserves</th>
<th>Risk margin</th>
<th>Best estimate</th>
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<tbody>
<tr>
<td></td>
<td>Best estimate</td>
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</tbody>
</table>

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Required Risk Capital (RC)

- A(0)
- L(0)
- C(0)

IAA: total balance sheet requirement
RC

"risk adjusted value"

\[ t=1: C(1) \]
Add risk margin (RM) for entire settlement of insurance portfolio
Diversification

Diversification = spreading the portfolio over a variety of exposures (products, markets, legal entities, etc.), rather than only a few selected areas.

◊ The pooling of many independent individual risks results in a low coefficient of variation of the P&L.

◊ On a larger scale, independent risk types (such as market and technical insurance risks) have a statistically compensatory effect on the relative P&L variability.

◊ ALM, Hedging: compensation of opposite effects of risk factors by opposite portfolio sensitivities.
Stochastic Factor Model

\[ C(1) = F(X) \]

1. Identification of relevant risk drivers \( X = (X_1, \ldots, X_n) \)

2. Sensitivity analysis:
   - “delta”: \( \delta_i F(x) = F(x + \epsilon_i) - F(x) \approx \partial_i F(x) \)
   - “gamma” or scenario vector for non-linear instruments

3. Model joint distribution of \( X = (X_1, \ldots, X_n) \)

4. \( \Delta C = C(1) - C(0) = F(X) - F(x) \)

5. Risk measurement \( \rho(\Delta C) \)
Covariance Model

\[ \Delta C = \nabla F(x) \cdot (X - x) =: Z_1 + \cdots + Z_n \]

- \( Z_1, \ldots, Z_n \) multi-normal distributed
- Risk measure affine in \( \sqrt{\text{Var}} \) (e.g. V@R or ES for normal d.)
  \[ \rho(Y) = \kappa \sqrt{\text{Var}(Y)} + \mathbb{E}[Y] \]
- Allocation to risk factors via marginal method
  \[ k_i = \frac{d}{d\epsilon} \rho(\Delta C + \epsilon Z_i) |_{\epsilon=0} = \kappa \frac{\sum_{j=1}^{n} \text{Cov}(Z_i, Z_j)}{\sqrt{\text{Var}(\Delta C)}} + \mathbb{E}[Z_i] \]
- Limited suitability for heavy tailed risk drivers and non-linear instruments
## Industry Practice: Covariance Method (example)

<table>
<thead>
<tr>
<th>Risk type</th>
<th>Geography A</th>
<th>Geography B</th>
<th>…</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>k</td>
<td></td>
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<tr>
<td>Credit</td>
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<td>Non-life Insurance</td>
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<tr>
<td>Life Insurance</td>
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</table>

**Level 1: within risk types**

\[ k = \kappa \sqrt{Var(Z)} + E[Z] \]

(CRO Forum: “A framework for incorporating diversification in the solvency assessment of insurers”)

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<tr>
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<td>BU2</td>
<td>BU2</td>
<td>BU2</td>
</tr>
<tr>
<td>Non-life Insurance</td>
<td>BU1</td>
<td>BU1</td>
<td>BU1</td>
</tr>
<tr>
<td>Life Insurance</td>
<td>BU2</td>
<td>BU2</td>
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</table>

Level 2: across risk types within BUs

\[
k = \kappa \frac{\sum_{BU1} \text{Cov}(Z, Z_i)}{\sqrt{\text{Var} \left( \sum_{BU1} Z_i \right)}} + \text{E}[Z]
\]

(CRO Forum: “A framework for incorporating diversification in the solvency assessment of insurers”)

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<td></td>
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</table>

Level 3: across BUs within geography

\[
k = \kappa \sqrt{\frac{\sum \text{Cov}(Z, Z_i)}{\text{Var} \left( \sum_{\text{Geography }A} Z_j \right)}} + E[Z]
\]

(CRO Forum: “A framework for incorporating diversification in the solvency assessment of insurers”)
Industry Practice: Covariance Method (example)

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Level 4: across geographies or regulatory jurisdictions

\[
k = \kappa \frac{\sum_{Group} Cov(Z, Z_i)}{\sqrt{\text{Var} \left( \sum_{Group} Z_i \right)}} + \text{E}[Z]
\]

(CRO Forum: “A framework for incorporating diversification in the solvency assessment of insurers”)
Problems

• Covariance method does not account for “tail dependence” (→ stress test, copulas!)

• Covariance capital allocation is not “fair”

<table>
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<th>Example: 3 BUs</th>
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<th>B</th>
</tr>
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<tbody>
<tr>
<td>BU1, BU2, BU3</td>
<td>k1, k2, k3</td>
<td></td>
</tr>
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\[
\text{correlation} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}
\]

Level 1 (stand alone): \( k_1 = k_2 = k_3 = 100 \)

Level 3 (within geography A): \( k_1 = \frac{1+0}{\sqrt{1+1}} \times 100 = 70.71 \)

Level 4 (full diversification): \( k_1 = \frac{1+1}{\sqrt{1+1+1+2 \times 1}} \times 100 = 89.44 \)
Problems

• Diversification effects require full fungibility of capital:

Regulatory risk: regulators may prevent capital to be transferred between jurisdictions

Management risk: companies’ managements may refuse to provide necessary capital injections

→ need of standardization for risk & capital transfers!
Standardized Diversification. An Example

- \( n \) BUs with (random) future available capitals \( C_1, \ldots, C_n \)
- Each BU faces a minimum capital requirement \( m_1, \ldots, m_n \)
- All BUs can share excess gain \((C_i-m_j)^+\) of BU \( i \) at appropriate price
- New positions of BUs become

\[
C_i + \sum_{j} \lambda_{ij} (C_j - m_j)^+ + \gamma_i
\]
Optimization Problem

\[ \inf_{\lambda_{ij}, \gamma_i} \sum_{i=1}^{n} \rho \left( C_i + \sum_{j=1}^{n} \lambda_{ij} (C_j - m_j)^+ + \gamma_i \right) \]

subject to the (sub-)clearing condition

\[ \sum_{i=1}^{n} \left( C_i + \sum_{j=1}^{n} \lambda_{ij} (C_j - m_j)^+ + \gamma_i \right) \leq \sum_{i=1}^{n} C_i \]

Fact: “<” may happen for some \( \omega \rightarrow \) dividends, group benefit

- Characterization of optimal allocation \( \lambda_{ij}, \gamma_i \)?
- Fair value for surplus participation \( (C_j - m_j)^+ \)?
- Extended formalization of this optimization problem?
Equilibrium Approach (Joint with Michael Kupper)

$n$ agents with initial endowments $X_i \in L^\infty = L^\infty(\Omega, \mathcal{F}, \mathbb{P})$ and convex risk measures $\rho_i : L^\infty \to \mathbb{R}$. Write $X := \sum_i X_i$.

1. monotone: $\rho_i(Z) \leq \rho_i(Y)$ if $Z \geq Y$
2. convex: $\rho_i(\lambda Z + (1 - \lambda)Y) \leq \lambda \rho_i(Z) + (1 - \lambda)\rho_i(Y)$, $\lambda \in [0, 1]$
3. cash invariant: $\rho_i(Z + \gamma) = \rho_i(Z) - \gamma$ for all $\gamma \in \mathbb{R}$
4. (coherent: $\rho_i(\lambda Y) = \lambda \rho_i(Y)$ for all $\lambda \geq 0$.)

Unconstrained optimization is trivial if $\rho_i \equiv \rho$ coherent:

$$\sum_{i=1}^{n} \rho(\lambda_i X + \gamma_i) = \rho(X) \leq \sum_{i=1}^{n} \rho(\xi_i) \quad \forall \sum_{i=1}^{n} \xi_i \leq X \quad \text{(sub-clearing)}$$

where $\lambda_i \geq 0$, $\sum_i \lambda_i = 1$ (convex risk sharing), $\sum_i \gamma_i = 0$ (cash rebalancing)

s.t. $\rho(\lambda_i X + \gamma_i) \leq \rho(X_i)$ (individual rationality)
Capital and Risk Transfer Constraints

Agent $i$ can only assume positions in $M_i \subset L^\infty$. $X_i \in M_i$.

**Assumption:** $M_i$ closed, convex, cash invariant: $M_i + \mathbb{R} = M_i$ (cash is fungible)

The $(M_i)$-constrained convolution of $\rho_1, \ldots, \rho_n$ is defined by

$$\square_i^{(M_i)} \rho_i(X) := \inf_{\sum_i \xi_i \leq X, \xi_i \in M_i} \sum_{i=1}^n \rho_i(\xi_i).$$

**Remark:** the “sub-clearing condition” $\sum_i \xi_i \leq X$ is essential!

**Assumption:** $\square_i^{(M_i)} \rho_i(X) > -\infty$

**Proposition 1.** $\square_i^{(M_i)} \rho_i : L^\infty \to \mathbb{R}$ is a convex risk measure with “penalty fct”

$$\alpha^{(M_i)}(\mu) = \sum_{i=1}^n \sup_{\xi_i \in M_i} (\langle \mu, -\xi \rangle - \rho_i(\xi)), \quad \mu \in (L^\infty)^*.$$ 

Moreover,

$$\emptyset \neq \partial \square_i^{(M_i)} \rho_i(\xi) \subset (-\mathcal{P}), \quad \forall \xi \in L^\infty$$

where $\mathcal{P} := \{ \mu \in (L^\infty)^* \mid \langle \mu, 1 \rangle = 1, \mu \geq 0 \} = \text{set of pricing rules.}$
Sub-Gradient

Let \( f : L^\infty \to \mathbb{R} \cup \{+\infty\} \) be convex. The sub-gradient is

\[
\partial f(\xi) := \{ \nu \in (L^\infty)^* \mid f(\eta) \geq f(\xi) + \langle \nu, \eta - \xi \rangle \ \forall \eta \in L^\infty \}\]

Example: \( f(\xi) = \rho_i^{M_i}(\xi) := \begin{cases} 
\rho_i(\xi), & \xi \in M_i \\
+\infty, & \text{else}, \end{cases} \)
Technical Results

Proposition 2. $\rho_i^{M_i} : L^\infty \to \mathbb{R} \cup \{+\infty\}$ is convex, cash invariant, $\sigma(L^\infty, (L^\infty)^*)$-lower semi-continuous and can be represented by

$$\rho_i^{M_i}(\xi) = \max_{\mu \in P} (\langle \mu, -\xi \rangle - \alpha_i^{M_i}(\mu)), \quad \forall \xi \in M_i$$

with "penalty function" $\alpha_i^{M_i}(\mu) := \sup_{\xi \in M_i} (\langle \mu, -\xi \rangle - \rho_i(\xi))$.

Moreover,

$$\inf_{\langle \mu, \eta \rangle \leq \langle \mu, \xi \rangle} \rho_i^{M_i}(\eta) = \langle \mu, -\xi \rangle - \alpha_i^{M_i}(\mu), \quad \forall \mu \in (L^\infty)^*, \xi \in M_i.$$

And the following statements are equivalent:

1. $-\mu \in \partial \rho_i^{M_i}(\xi)$
2. $\rho_i^{M_i}(\xi) = \langle \mu, -\xi \rangle - \alpha_i^{M_i}(\mu)$
3. $\inf_{\langle \mu, \eta \rangle \leq \langle \mu, \xi \rangle} \rho_i^{M_i}(\eta) = \rho_i^{M_i}(\xi)$

Corollary 3. $\partial \rho_i^{M_i}(\xi) \cap (-P) \neq \emptyset, \forall \xi \in M_i.$
Equilibrium and Optimality

An allocation \( \xi_1, \ldots, \xi_n \) is **attainable** if \( \xi_i \in M_i \) and \( \sum_i \xi_i \leq X \).

An attainable allocation \( \xi_1, \ldots, \xi_n \) is Pareto **optimal** if

\[
\rho_i(\eta_i) \leq \rho_i(\xi_i) \quad \forall i \quad \Rightarrow \quad \rho_i(\eta_i) = \rho_i(\xi_i) \quad \forall i
\]

for every attainable allocation \( \eta_1, \ldots, \eta_n \).

An attainable allocation \( \xi_1, \ldots, \xi_n \) together with a pricing rule \( \mu \in \mathcal{P} \) is an Arrow–Debreu **equilibrium** if

\[
\langle \mu, \xi_i \rangle \leq \langle \mu, X_i \rangle \quad \text{and} \quad \rho_i(\xi_i) = \inf_{\langle \mu, \eta \rangle \leq \langle \mu, X_i \rangle, \eta \in M_i} \rho_i(\eta) \quad \forall i.
\]
Characterization of Optimality

Let $\xi_1, \ldots, \xi_n$ be an attainable allocation, $\xi_0 := X - \sum_i \xi_i \geq 0$. The following are equivalent:

1. $\xi_1, \ldots, \xi_n$ is optimal

2. $\Box_i^{(M_i)} \rho_i(X) = \sum_{i=1}^n \rho_i(\xi_i)$

3. $\langle \mu, \xi_0 \rangle = 0$ (price clearing) and $-\mu \in \partial \rho_1^{M_1}(\xi_1) \cap \ldots \cap \partial \rho_n^{M_n}(\xi_n)$ for some $\mu \in \mathcal{P}$.

Practical implementation: minimize 2. Or find allocations $\xi_1, \ldots, \xi_n$ such that $\cap_i \partial \rho_i^{M_i}(\xi_i)$ is not empty. Check attainability and $\langle \mu, \xi_0 \rangle = 0$. 
Welfare Theorems

$\xi_1, \ldots, \xi_n, \mu$ is an equilibrium if and only if

1. $\xi_1, \ldots, \xi_n$ is optimal and

2. $-\mu \in \partial \Box_i^{(M_i)} \rho_i(X)$ (optimal pricing rules) and

3. $\langle \mu, \xi_i \rangle = \langle \mu, X_i \rangle$ for all $i$.

Moreover, if $\xi_1, \ldots, \xi_n$ is optimal and $-\mu \in \partial \Box_i^{(M_i)} \rho_i(X)$ then

$$\xi_1 + \langle \mu, X_1 - \xi_1 \rangle, \ldots, \xi_n + \langle \mu, X_n - \xi_n \rangle$$

is an equilibrium.
Approximate Equilibrium

Optimal allocations do not always exist! BUT:

Let \(-\mu \in \partial \square_i^{(M_i)} \rho_i(X) \neq \emptyset, c(-p)\) and \(\epsilon > 0\). Then there exists an attainable allocation \(\xi_1, \ldots, \xi_n\) such that:

1. \(\square_i^{(M_i)} \rho_i(X) \geq \sum_{i=1}^{n} \rho_i(\xi_i) - n\epsilon\) (“\(\epsilon\)-optimality”) and

2. \(\langle \mu, -X_i \rangle - \alpha_i^{M_i}(\mu) \geq \rho_i(\xi_i) - \epsilon\) (“\(\epsilon\)-equilibrium”) and

3. \(\langle \mu, X_i \rangle - \epsilon \leq \langle \mu, \xi_i \rangle \leq \langle \mu, X_i \rangle\) for all \(i\).

Practical limitations: optimal pricing rules \(\partial \square_i^{(M_i)} \rho_i(X)\) may be difficult to find.
Special Case

Let $Z_0, \ldots, Z_m \in L^\infty$ be linearly independent, with $Z_0 \equiv 1$ ("cash").

Assumptions:

- $X_i$ and $Z_0, \ldots, Z_m$ linearly independent for all $i$
- $M_i = \left\{ X_i + \sum_{j=0}^{m} \lambda_j Z_j \mid \lambda_j \in \mathbb{R} \right\}$ finite-dimensional affine spaces
- $v_i(\lambda_0, \ldots, \lambda_m) := \rho_i(X_i + \sum_{j=0}^{m} \lambda_j Z_j)$ differentiable in $\lambda_j$

Then

1. $v_i(\lambda_0, \ldots, \lambda_m) = v_i(0, \lambda_1, \ldots, \lambda_m) - \lambda_0 =: u_i(\lambda_1, \ldots, \lambda_m) - \lambda_0$

2. $\nu \in \partial \rho_i^M(X_i + \sum_j \lambda_j Z_j) \Leftrightarrow \langle \nu, 1 \rangle = -1$ and $\langle \nu, Z_j \rangle = \partial_{\lambda_j} u_i(\lambda_1, \ldots, \lambda_m) \forall j \geq 1$

$\rightarrow$ Equilibrium pricing rule (if it exists) does not interfere with any a priori values of $X_1, \ldots, X_n$!
The clearing condition

$$\sum_{i=1}^{n} \left( X_i + \sum_{j=0}^{m} \lambda_{ij} Z_j \right) \leq X \iff \sum_{j=0}^{m} \left( \sum_{i=1}^{n} \lambda_{ij} \right) Z_j \leq 0$$

is always satisfied (“and vice versa”) for

$$\sum_{j=1}^{m} \left( \sum_{i=1}^{n} \lambda_{ij} \right) Z_j - \text{ess sup} \left( \sum_{j=1}^{m} \sum_{i=1}^{n} \lambda_{ij} Z_j \right)_{\text{net cash}}$$

→ \((m \times n)\)-dimensional optimization problem

$$\Box_{(M^i)}^{(M^i)} \rho_t (X) = \inf_{\lambda_0} \left( \sum_{j=1}^{m} \sum_{i=1}^{n} u_i (\lambda_{i1}, \ldots, \lambda_{im}) + \text{ess sup} \left( \sum_{j=1}^{m} \sum_{i=1}^{n} \lambda_{ij} Z_j \right) \right)$$
Characterization of Optimum

Corollary. The optimum is attained:

$$\square_i^{(M_i)} \rho_i(X) = \sum_{j=1}^{m} \sum_{i=1}^{n} u_i(\lambda_{i1}^*, \ldots, \lambda_{im}^*) + \text{ess sup} \left( \sum_{j=1}^{m} \sum_{i=1}^{n} \lambda_{ij}^* Z_j \right)$$

If and only if

1. $\partial_{\lambda_i} u_i(\lambda_{i1}^*, \ldots, \lambda_{im}^*) = \partial_{\lambda_i} u_1(\lambda_{i11}^*, \ldots, \lambda_{i1m}^*) = : \langle \mu, -Z_j \rangle \ \forall i, j$ and

2. $\sum_{j=1}^{m} \sum_{i=1}^{n} \lambda_{ij}^* \langle \mu, Z_j \rangle = \text{ess sup} \left( \sum_{j=1}^{m} \sum_{i=1}^{n} \lambda_{ij}^* Z_j \right)$ (price clearing)

An equilibrium allocation is then obtained by

$$X_i + \sum_{j=1}^{m} \lambda_{ij}^* Z_j - \sum_{j=1}^{m} \lambda_{ij}^* \langle \mu, Z_j \rangle$$

and $\langle \mu, Z_j \rangle$ are the fair prices for the risk and capital transfers $Z_j$
Concrete Example

\( m = n = 2 \) insurance units. \( Z_i = (C_i - m_i)^+ \) surplus participation.

\((C_1, C_2) = \text{random (100pt) sample of } N\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \sigma^2 \begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix}\right)\)

Assumption: available \((= C_i(0) = 1) = \text{required capital:}\)

\[ 0 = \text{ES}(C_i) = \sigma \text{ES}(N(0, 1)) - 1 \iff \sigma = \frac{1}{\text{ES}(N(0, 1))} = 0.375204, \]

for confidence level \( \alpha = 0.01. \)
ES\( (C_1) = -0.175 \), ES\( (C_2) = -0.171 \), ES\( (C_1 + C_2) = -0.850 \).

Minimum capital \( m_i := 0.4 \times (1 + \text{ES}(C_i)) \) (=SST risk margin)
\( m_1 = 0.330 \), \( m_2 = 0.332 \).

Optimal allocation \( (\lambda^*_1, \lambda^*_2) = (-0.58, 0.75), (\lambda^*_{21}, \lambda^*_{22}) = (0.50, -0.81) \).

Net cash flow = 0.01,

fair values: \( \langle \mu, (C_1 - m_1)^+ \rangle = 0.04 \), \( \langle \mu, (C_2 - m_2)^+ \rangle = 0.15 \).

ES\( (C_1 - 0.58 \times ((C_1 - m_1)^+ - 0.04) + 0.75 \times ((C_2 - m_2)^+ - 0.15)) = -0.37 \)

ES\( (C_2 + 0.50 \times ((C_1 - m_1)^+ - 0.04) - 0.81 \times ((C_2 - m_2)^+ - 0.15)) = -0.48 \)

sum = -0.85 ⇒ full diversification effect (in general not)
Summary

We propose an equilibrium approach to valuate and measure contingent capital and risk transfers between insurance units in order to account for diversification benefits.

The available capital does not change (equilibrium) while the maximal reduction of required capital is achieved (optimum).

The first example with surplus participations $Z_i = (C_i - m_i)^+$ can be generalized. Results are available for arbitrary convex and cash invariant sets $M_i \subset L^\infty$ of capital and risk transfers (D.F. and M.Kupper: “Equilibrium and optimality for monetary utility functions under constraints”)

Also the framework $L^\infty$ can be extended ($L^\infty$ does not contain normal distributed rvs.). This is work in progress (joint with Michael Kupper and Gregor Svindland, University of Munich).