Exotic Options
Through 100 Years to Current State of the Art
Risk Management of Investment Guarantees
Contents

- The First 100 Years
- Exotic Options
- State of the Art Risk Management
The First 75 Years

- Bachelier (1900)
- Levy (1925)
- Ito (1951)
- Markowitz (1952)
- Reddington (1952)
- Sprenkle (1958)
- Samuelson (1965)
- Thorpe (1969)
- Black & Scholes (73)
Black & Scholes

- Black & Scholes (1973)
- Merton (1973)
- Harrison & Kreps (1977)
Exotic Options
Exotic Options

- Margrabe (1978)
  \[ P_O_T = \max(Q_1 S_1(T) - Q_2 S_2(T), 0) \]

  \[ P_O_T = \max(\max(S_1, S_2) - X, 0) \]

Deferred Annuities

- Margrabe (1978)

\[ P_{OT} = \max(Q_1 S_1(T) - Q_2 S_2(T), 0) \]
Deferred Annuities

- Margrabe (1978)

\[ PO_T = \max(Q_1 S_1(T) - Q_2 S_2(T), 0) \]

\[ GC_T = \max( g b_T a_{x+T,i(T)} - A S_T, 0) \]
Compound Options


\[ PO_T = \max(\max(S_1, S_2) - X, 0) \]
Compound Options


\[ PO_T = \max(\max(S_1, S_2) - X, 0) \]

- Useful for valuing benefits of the form

\[ \max(S_1, S_2, X) = X + \max(\max(S_1, S_2) - X, 0) \]

- Generalises

\[ \max(S_1, S_2, S_3, X) = \max(S_1, \max(S_2, S_3, X, 0), 0) \]
Guaranteed Annuity Options


\[ P_{OT} = \max(\max(S_1, S_2) - X, 0) \]

\[ \text{Benefit} = \max(AS_T, GB_T, GB_T g_T a_{x+T, i(T)}) \]

\[ GC_T = \max(AS_T, GB_T, GB_T g_T a_{x+T, i(T)}) - AS_T \]

\[ = GB_T + \max(\max(AS_T, GB_T g_T a_{x+T, i(T)}) - GB_T, 0) - AS_T \]

\[ ULBenefit = \max(AS_T, AS_T g_T a_{x+T, i(T)}) \]

\[ CommuteGC_T = \max(AS_T, gb_T a_{x+T, i(T)}, gb_T h_T) - AS_T \]
Regular Premium Business

- **Asian Options**
  Options based on the average rather than the terminal value of the underlying asset/index

- **Regular Premium**
  Policyholder has an option to pay future premium instalments -- office

\[
AS_T = (1 - w)^T (1 - k) P \left[ \frac{I_T}{I_0} + \frac{I_T}{I_1} + \cdots + \frac{I_T}{I_{T-1}} \right]
\]

\[
AS_T = (1 - w)^T (1 - k) P [S_1 + S_2 + \cdots S_T]
\]

\[
= (1 - w)^T (1 - k) P T \bar{S}_T
\]

\[
GC_T = \max(GB_T - AS_T, 0)
\]

\[
= \max(GB_T - (1 - w)^T (1 - k) T \bar{P} \bar{S}_T, 0)
\]
State of the Art
Risk Management
Economic Balance Sheet
Unhedged

<table>
<thead>
<tr>
<th>Starting Position</th>
<th>Total Liabs</th>
<th>Total Assets</th>
<th>Equity</th>
<th>Bonds</th>
<th>Derivatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset Backing</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Value of Guarantee</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Balance</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>125</td>
<td>125</td>
<td>62.5</td>
<td>62.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equities down 20%</th>
<th>Total Liabs</th>
<th>Total Assets</th>
<th>Equity</th>
<th>Bonds</th>
<th>Derivatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset Backing</td>
<td>90</td>
<td>90</td>
<td>40</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Value of Guarantee</td>
<td>25.6</td>
<td>18</td>
<td>8</td>
<td>10</td>
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</tr>
<tr>
<td>Balance</td>
<td>-3.1</td>
<td>4.5</td>
<td>2</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>112.5</td>
<td>112.5</td>
<td>50</td>
<td>62.5</td>
<td></td>
</tr>
</tbody>
</table>

- Scenario: equities down 20%, bonds unchanged
- 20% equity fall → value of guarantee increases to 25.6
- ALM capital impact: £7.6. Difference between new value of guarantees and re-valued assets that support guarantees (= 25.6–18)
- Capital injection = £3.1 to restore solvency and £7.6 to restore financial strength to perfectly matched position
### Economic Balance Sheet
#### OTC Hedge

<table>
<thead>
<tr>
<th>Starting Position</th>
<th>Total Liabs</th>
<th>Total Assets</th>
<th>Equity</th>
<th>Bonds</th>
<th>Derivatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset Share</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Value of Guarantee</td>
<td>20</td>
<td>20</td>
<td></td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Balance</td>
<td>5</td>
<td>5</td>
<td>2.5</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>125</td>
<td>125</td>
<td>52.5</td>
<td>52.5</td>
<td>20</td>
</tr>
</tbody>
</table>

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<th>Equity</th>
<th>Bonds</th>
<th>Derivatives</th>
</tr>
</thead>
<tbody>
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<td>Asset Share</td>
<td>90</td>
<td>90</td>
<td>40</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Value of Guarantee</td>
<td>25.6</td>
<td>25.6</td>
<td></td>
<td></td>
<td>25.6</td>
</tr>
<tr>
<td>Balance</td>
<td>4.5</td>
<td>4.5</td>
<td>2</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>120.1</td>
<td>120.1</td>
<td>42</td>
<td>52.5</td>
<td>25.6</td>
</tr>
</tbody>
</table>

- Buy an OTC put option to match the liability guarantee
- In theory a perfect match: no ALM impact or capital injection; but:
  - Pricing & Cash Flow
  - At mercy of market: pay implied volatility of market / counterparty
  - Other assumptions needed to make hedge perfect
  - Periodic rebalancing required, incurs high transactions costs
Replication

Put Option = (Short Position in Underlying) 
+ Long Position in Risk Free 
+ Position in Volatility Sensitive Asset 
+ Position in Interest Sensitive Asset

- Delta: protects against small immediate changes in underlying
- Gamma: protects against small immediate changes in delta
- Vega: protects against small immediate changes in Vol
- Rho: protects against small immediate changes in interest
Dynamic Hedge in the Balance Sheet

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Bonds</th>
<th>Risk Free</th>
<th>Options</th>
<th>Swaps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset Backing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value of Guarantee</td>
<td>20</td>
<td>(24)</td>
<td>(24)</td>
<td>60</td>
<td>5</td>
</tr>
<tr>
<td>Balance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- “Manufacture” the OTC derivative
- Protects against risks of:
  - Underlying assets falling
    - Volatility spiking
    - Interest rates changing
- Dynamic ➔ Technology requirements
Risk Inventory

- INVESTMENT RISK
  - Market
  - Interest Rate
  - Credit

- DEMOGRAPHIC RISK
  - Underwriting
  - Experience Statistics
  - Reinsurance

- OPERATIONAL RISK
  - Procedures and Controls
  - Mis-selling
Dynamic Hedging: Requirements

- Market consistent stochastic valuation of liability guarantees on a policy-by-policy and daily basis
- Capital markets expertise to identify hedge portfolio
- Systems capability to dynamically monitor position frequently
- Effective control systems
  - Performance and risk attribution through analysis of profit movements
  - Financial reporting requirements
  - Financial projection analysis – back testing of strategy to demonstrate effectiveness of the hedge

- The bottom line: a very complex task requiring specialist actuarial, capital market and systems expertise / investment
MG-Hedge System
structure reflects market risk management framework

- Actuarial Liability Valuation
  - Per policy seriatim
  - Stochastic

- Trade Positioning System
  - Live market data
  - Asset management interface

- Financial Reporting System
  - Financial control
  - Profit measurement, analysis and projections

IT Interface
- In force Data
- IT Interface
- Asset & Market Data
Overview of Hedging Program

Daily
- Trading Grid
- Nightly Valuation
- Monitor Markets

Weekly
- Produce P&L Report
- Performance Attribution

Quarterly/Annually
- Monitor Experience
  - Fund Modeling
  - Mortality
  - Lapses
  - Fund Transfers
- Financial Reporting
- Hedging Strategy for New Products
**Goal of Hedging**

Replicate Embedded Option So That:

<table>
<thead>
<tr>
<th>Beginning of Period Guarantee Value</th>
<th>Beginning of Period Hedge Asset Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ Interest</td>
<td>+ Gains on Hedge Assets due to market movements</td>
</tr>
<tr>
<td>- Claims</td>
<td>- Losses on Hedge Assets due to market movements</td>
</tr>
<tr>
<td>+ Guarantee Premiums</td>
<td></td>
</tr>
<tr>
<td>+ Changes due to market movements</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>End of Period Guarantee Value</th>
<th>End of Period Hedge Asset Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{EOP Guarantee Value} \quad \text{EOP Hedge Asset Value} \]

\[ \text{Net Gain (Loss)} \]

\[ \text{Balance Sheet Volatility} \]

Allowing for Demographic & Path Dependencies
## Income Statement Projection

<table>
<thead>
<tr>
<th></th>
<th>Projection Year</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total Income</strong></td>
<td>$12,274.0</td>
<td>$46,955.1</td>
<td>$96,128.8</td>
<td>$124,681.3</td>
<td>$7,796.7</td>
</tr>
<tr>
<td><strong>Premium Income</strong></td>
<td>$668.4</td>
<td>$1,728.0</td>
<td>$3,461.3</td>
<td>$2,901.5</td>
<td>$2,608.6</td>
</tr>
<tr>
<td><strong>Investment Income</strong></td>
<td>$11,605.6</td>
<td>$45,227.0</td>
<td>$94,667.5</td>
<td>$121,779.8</td>
<td>$5,183.1</td>
</tr>
<tr>
<td><strong>Fixed Income Portfolio</strong></td>
<td>$244.0</td>
<td>$1,470.2</td>
<td>$4,729.2</td>
<td>$10,155.3</td>
<td>$13,586.5</td>
</tr>
<tr>
<td><strong>Futures</strong></td>
<td>$1,864.6</td>
<td>$6,156.0</td>
<td>$8,421.2</td>
<td>$6,260.8</td>
<td>$333.8</td>
</tr>
<tr>
<td><strong>Options &amp; Swaps</strong></td>
<td>$9,497.0</td>
<td>$37,600.8</td>
<td>$81,517.1</td>
<td>$105,363.7</td>
<td>$(8,732.2)</td>
</tr>
<tr>
<td><strong>Total Expenses</strong></td>
<td>$9,826.8</td>
<td>$39,441.3</td>
<td>$91,189.1</td>
<td>$125,981.0</td>
<td>$11,270.7</td>
</tr>
<tr>
<td><strong>Increase in Fair Value Liability</strong></td>
<td>$9,758.1</td>
<td>$39,291.6</td>
<td>$91,067.3</td>
<td>$125,977.3</td>
<td>$11,270.7</td>
</tr>
<tr>
<td><strong>Interest on Debt</strong></td>
<td>$68.6</td>
<td>$149.7</td>
<td>$121.8</td>
<td>$3.7</td>
<td>-</td>
</tr>
<tr>
<td><strong>Pre-Tax Income</strong></td>
<td>$2,447.2</td>
<td>$7,513.8</td>
<td>$6,939.7</td>
<td>$(1,299.7)</td>
<td>$(3,474.0)</td>
</tr>
<tr>
<td><strong>Equity Market Return</strong></td>
<td>-25%</td>
<td>-25%</td>
<td>-25%</td>
<td>-25%</td>
<td>15%</td>
</tr>
<tr>
<td><strong>10 Year Interest Rate</strong></td>
<td>5%</td>
<td>5%</td>
<td>4%</td>
<td>4%</td>
<td>3%</td>
</tr>
</tbody>
</table>
Hedge Reporting
P&L - Unhedged vs. Hedged
## Replication

<table>
<thead>
<tr>
<th><strong>Reddington Immunisation</strong></th>
<th><strong>Black Scholes Dynamic Hedging</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Immunises</td>
<td>Immunises</td>
</tr>
<tr>
<td>Small immediate changes</td>
<td>Small Immediate Changes</td>
</tr>
<tr>
<td>Frequent Rebalancing</td>
<td>Frequent Rebalancing</td>
</tr>
<tr>
<td>Duration Matching (rate of change wrt underlying interest)</td>
<td>Delta (rate of change of value with respect to underlying)</td>
</tr>
<tr>
<td>Convexity Matching (second derivative)</td>
<td>Gamma (second derivative)</td>
</tr>
<tr>
<td></td>
<td>ADD: Vega, Rho etc</td>
</tr>
</tbody>
</table>
For further information, please contact

Gary Finkelstein  
Milliman Financial Risk Management  
Finsbury Tower  
103-105 Bunhill Row  
London EC1Y 8LZ

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Email: gary.finkelstein@milliman.com