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An Investigation of the loss of an CAPM-portfolio

Considerations on risk, measurement of risk and safeguarding risk of investments

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This article is the opinion of the authors, not an official opinion of the Dr.Dr. Heissmann GmbH
Motivation: Risk of a pension fund

- Pension funds: Assets (e.g. shares, bonds) used for paying benefits
- Risk of a pension funds: Rate of return is too low to pay benefits; insolvency of funds or employer
- Germany: Funds or assets of employer are safeguarded by Pensionssicherungsverein auf Gegenseitigkeit PSVaG
- PSVaG demands a premium proportional to investment (book reserve) from all companies owing funds; pay as you go finance
- Wealthy companies pay for poor companies (insolvency)
- Thesis: With such a premium a prudent man will invest in the assets with highest rate of return not considering the risk of this investment
Risk of an investment: definition of the loss

- Loss of an investment: rate of return is below an expected value $R_{\text{neq}}$
- Measures of loss:
  - Noise
  - Shortfall
  - Variance: (CAPM, (Sharpe, W.F. The Journal of Finance 19, 425-42 (1964))
  - Value at risk ($1-\alpha$ of all rates of return are larger than $R_{\text{VaR}}$)
  - All these measures give a value of the probability of a loss or a value of the maximal loss (with $1-\alpha$ confidence)

- Alternative measure?
Risk of an Investment: average loss $L_A$

- Definition of the average loss based on lower partial moments

\[ L_A = I \int_{-\infty}^{R_{\text{nec}}} (R_{\text{nec}} - R) p(R) dR \]

- $R_{\text{nec}}$: rate of return, which is necessary for the investor; $I$: amount of investment, $R$: rate of return of the investment, $p(R)$: probability density

- The probability to obtain a loss larger than 0 is:

\[ \int_{-\infty}^{R_{\text{nec}}} p(R) dR \]

with \[ \int_{-\infty}^{\infty} p(R) dR = 1 \]

- upper boundary $R_{\text{nec}}$ instead of oo, because we measure the loss, not the gain of the investment

- $L_A$ is a measure of the expected loss: i.e.

\[ L_A (R_1) > L_A (R_{\text{II}}) \]

means, that using portfolio I, a larger loss must be expected and has to be safeguarded than using portfolio II
Risk of an Investment: average loss $L_A$

- $L_A$ is a function of $R$, $R_{\text{nec}}$ and $p(R)$
- $L_A$ is not additive with respect to $R$, $p(R)$ and $R_{\text{nec}}$. 
- Using $L_A$:
  - measurement of $L_A$ of different investments or portfolios
  - optimization of the composition of different portfolios with respect to
    - average loss
    - expected rate of return, when the average loss can be safeguarded.
Average loss $L_A$ of a CAPM-portfolio

- CAPM portfolio consists of “bonds” and “shares”, (see cash and tangential portfolio of the CAPM theory)
- Bond: $\sigma = 0$, i.e. no variance, fixed rate of return $R_0$. Shares: $\sigma_{TP} > 0$, variable rate of return $R_{TP}$
- The rate of return of this (pension fund) portfolio is:

$$R_{PF} = \alpha R_0 + (1-\alpha)R_{TP}$$

and the expected rate of return

$$\mu_{PF} = \alpha \mu_0 + (1-\alpha)\mu_{TP}$$

and the standard deviation

$$\sigma_{PF} = (1-\alpha)\sigma_{TP}$$

with $\mu_{TP}$ the expected rate of return. We assume $0 \leq \alpha \leq 1$ (investment on the capital market line).
Average loss $L_A$ of a CAPM-portfolio

- We obtain for $L_A$:

$$L_A = \int_{-\infty}^{\infty} (R_{nec} - \alpha R_0 - (1 - \alpha)R_{TP}) p(\alpha R_0 + (1 - \alpha)R_{TP}) d(\alpha R_0 + (1 - \alpha)R_{TP})$$

we define:

$$p_{PF}(R_{TP}) \equiv p(\alpha R_0 + (1 - \alpha)R_{TP})$$

We will use only $p_{PF}(R_{TP})$ and we call this $p(R_{TP})$. We conclude:

$$L_A = I(1 - \alpha) \int_{-\infty}^{\alpha R_0} (R_{nec} - \alpha R_0 - (1 - \alpha)R_{TP}) p(R_{TP}) dR_{TP}$$

- $L_A$ can not been calculated straight forward, neither the derivate of $L_A$ as function of $\alpha$

- Assuming that $R_{TP}$ is normally distributed, we can calculate upper limits for $L_A$ in an analytic manner

- For $R_0 > R_{nec}$, there is a minimum of $L_A$ at $\alpha = 1$
Average loss of a CAPM-portfolio. Example MSCI

- Shares described by MSCI-Europe-ET-performance-index
- Assumptions: rate of return is normally distributed, investment period of 1 year:
  - average rate of return of 10.8 % and variance of 20 %; investment period of 1 year
    (average rate of return of 0.64% and variance of 7.90 %; investment period of 1 month)
- $R_{\text{neg}} = 6 \%$ and $3.25\%$; $R_0 = 3 \%$, or $1.5 \%$
- $L_A$ as a function of the composition of the portfolio is not monotone, but has a minimum for $\alpha = 0.8$
Fig. 1 $L_A$ of a CAPM-portfolio (shares described by MSCI EUROPE Performance Index)
Safeguarding of the loss and optimization of an CAPM-portfolio

- We undewrite the average loss by an insurance.
- Concerning the premium of the insurance contract, we investigate three scenarios:
  - A: the premium is proportional to the investment: \( P = \pi I \)
  - B: the premium is proportional to the investment in shares: \( P = \pi I (1-\alpha) \)
  - C: the premium is proportional to the loss: \( P = \pi L_A \)
- We define an utility function \( U \) as sum of the expected return of the investment minus the premium.

\[
U = I \left( aR_0 + (1-\alpha)R_{TP} \right) - P
\]

- The investor builds his portfolio, that he obtains the maximum in the utility function.
Safeguarding of the loss of a portfolio and building of the portfolio

- Scenario A: \( P = \pi I \):
  \[
  U = I (aR_0 + (1-\alpha)R_{TP}) - I \pi = I (aR_0 + (1-\alpha)R_{TP} - \pi)
  \]
  For \( R_{TP} > R_0 \), the investor will always choose a portfolio which contains only shares.

- Scenario B:
  \[
  U = I (aR_0 + (1-\alpha)R_{TP}) - (1-\alpha)I \pi = I (aR_0 + (1-\alpha)(R_{TP} - \pi))
  \]
  An investor will invest in shares, when \((R_{TP} - \pi) > R_0\) and in bonds, when \((R_{TP} - \pi) < R_0\)

- Scenario C:
  \[
  U = I (\alpha R_0 + (1-\alpha)R_{TP}) - L_A \pi
  \]
  Maximization of \( U \) by variation of \( \alpha \) can only be done by numerical methods.

- different premiums \( \pi \), i.e. 0.1, 0.5, 1 and 2

- For \( \pi > 1 \), local maximum for \( 0 < \alpha < 1 \)
Figure 2: Utility function of a CAPM portfolio, if the average loss $L_A$ is safeguarded by different insurance premiums, $R_{\text{net}} 06\%$, $R_0 = 3\%$
Conclusion

- Average loss $L_A$ is appropriate to measure the loss of a portfolio
- Using the average loss $L_A$ we can optimize the portfolio with respect to $L_A$
- Calculating $L_A$ and safeguarding it, we can optimize the portfolio with respect to the utility function $U$
- $U$ is the expected return of the investment minus the insurance premium.
- Parameters and values, used to calculate the premium are important, for the portfolio composition, considering maximal values of $U$; i.e. maximal rate of return and no loss.