Inflation and Excess Insurance

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Abstract

We study the effect of inflation on insurance covers with retention, e.g. insurance policies with a deductible or reinsurance layers. If we have an inflation of say 3% per year from the ground up, what is the resulting impact on the insured losses? On the contrary, if we observe a rising loss burden affecting an insurance product, can we deduct from this the inflation from the ground up?

We will see that the answers to these questions depend heavily on the tail of the loss size distribution – which is an important source of uncertainty as we might not exactly know this tail.

The tail-dependency of inflationary effects helps further understand why inflation is not perceived as a major issue in some insurance lines, while being a dramatic problem in other areas, in particular for MTPL reinsurance layers.

Keywords

Inflation, reinsurance, excess insurance, layer, leverage effect, GPD, Pareto
Introduction

Deductibles (retentions, attachment points) are quite common in insurance, ranging from about 100 Euro in Personal Lines, e.g. Motor Own Damage, to 100,000 Euro and more in industrial insurance. Even higher, in the million Euro range, are the typical retentions of the so-called non-proportional reinsurance treaties. Although of very diverse character these insurance products have one common problem that shall be treated in this paper: If we have an inflation of say 3% per year affecting the insured losses f.g.u. (from the ground up), what is the resulting increase of the risk premium if the (re)insurance cover has a deductible?

The mathematics answering this question is easy and well known, just some algebra using the inflation rate and the loss size distribution, see e.g. Klugman et al. However it should be taken into account that in practice we have some uncertainty regarding this information:

Firstly we might not exactly know the loss size distribution. Data might be too few for a very reliable model selection and parameter estimation. Furthermore the data is typically incomplete in the area of small losses as the losses below the deductible are not, or not completely, reported to the insurer.

A possibly bigger problem is how to determine the correct inflation rate. Even in very developed insurance markets where many specific inflation indexes – consumer prices, construction cost, wages, etc. – are readily available it is somewhat uncertain whether such an index exactly reflects the inflation of the business.

So in many cases it would be good to estimate the inflation directly from the loss data, which leads to the converse of the above question: If we just have the loss data of the excess (re)insurance cover for different periods, can we estimate the ground-up inflation, i.e. the inflation of the losses f.g.u.?

In the following we will treat the first question. The somewhat surprising results lead to a (not too optimistic) view in respect of the second question.

The paper is organized as follows. Section 1 provides some basic math and an extreme example showing the variety of effects that inflation may have on excess insurance. Section 2, the core part of this paper, introduces inflation leverage effects, which facilitate a general and systematic study of the impact of inflation to the loss frequency and the loss severity of an insurance layer. Sections 3 and 4 apply these results to loss distributions with a Generalized Pareto tail, making clear the extreme tail-dependency of inflationary effects. The final section applies the developed results to two important reinsurance areas, one being highly sensitive to inflation, the other one not at all.

1. Basics

We use the common notation of the collective model of risk theory: Let $N$ be the number of losses in an insurance period. Let $X$ be the size of a loss f.g.u. with distribution function $F$, survival function $S=1−F$ and (if existent) density $f$.

Notice that this model is more general than it appears to be. The losses could be single ones or accumulation losses per (e.g. natural) event, the latter being the appropriate model for catastrophe reinsurance. Alternatively if the layer applies to the aggregate of all losses in a certain insurance period (like Stop-Loss reinsurance or e.g. some Health insurance products) then we let $X$ be this aggregate loss and set $N = 1$. 
If we compare two states of the world, one before and one after a certain inflation, the “world after” shall be denoted by a “∗”: X∗ would have the distribution F∗, etc.

Inflation numerically means multiplication of all money amounts by a factor g=1+h, with h being the inflation rate. Normally g ≥ 1 but deflation exists so all values g > 0 are admitted. (As usual we assume uniform inflation for the kind of losses we are modeling with X. If this assumption is not believed in it will be hard to do any modeling. However, segmentation of losses into arguably homogeneous loss types can be a solution for such cases, e.g. in the Third Party Liability lines one could model property damage and personal injury losses separately.)

As inflation only affects the loss sizes N∗ has the same distribution as N.

X∗ has the same distribution as gX, hence F*(x) = P(X* ≤ x) = P(gX ≤ x) = P(X ≤ x/g) = F(x/g)

Let c be the amount covered by the (re)insurance layer and d be the deductible. One writes shortly:

c xs d

In case X has finite expectation c may be infinite. Examples for this are certain Health policies and Personal Lines Third Party Liability in some countries.

If X is the original loss the “loss to the layer” (the part covered by the layer) equals min((X−d)+,c).

As this is an increasing (more accurately non-decreasing) function in X it is clear that with positive inflation the expectation of the layer loss will rise, and so will the loss frequency (more losses will exceed d).

For unlimited layers one has a stronger result. It is well known that

E((X∗−d)+) = E((gX−d)+) = g E((X−d/g)+) ≥ g E((X−d)+) for g ≥ 1

Thus here the effect of inflation on the expected value is more than proportionate.

We conclude this section with a maybe a bit weird example from reinsurance, which makes clear that while rising losses let the loss frequency and the expected loss increase nothing can be said in general about the impact of inflation to the severity of the layer.

Consider a reinsurance program with a retention of 1 (say million Euro). Let it be split into the three layers:

1 xs 1, 3 xs 2, 5xs 5

Let the loss size distribution before inflation be as follows:

- 97% of the losses are smaller than 0.8
- 2% of the losses equal 0.9
- 1% of the losses equal 5.5

Some time later, after 20% inflation, we have:

- 97% of the losses are smaller than 0.96
- 2% of the losses equal 1.08
- 1% of the losses equal 6.6
Before inflation 99% of the losses fall into the retention. Only if a loss equals 5.5 the reinsurance applies, resulting in a total loss to both the first and the second layer and a partial loss of 0.5 to the top layer.

After inflation we have a completely different situation:

- The upper layer is hit by the same losses as before (same loss frequency) but each of these now equals 1.6, which means that both the severity and the risk premium increase by 220%, which is definitely much more than proportionately.
- The medium layer experiences the same losses as before, being again total losses, so all parameters (frequency, severity, expectation) remain unchanged. 20% inflation had no effect at all to this layer.
- The situation in the bottom layer changes radically. The losses of the medium class now become excess losses, which results in a triplication of the loss frequency. With the old losses remaining total losses and the new ones amounting to 0.08 each, the average loss decreases drastically to 0.39. The risk premium increases by 16%, which is a bit less than proportionately.

From a practical viewpoint one could argue that this reinsurance program is unrealistic because it is inappropriate for the loss distribution. (The upper layer is largely unexposed.) But inappropriate programs occasionally occur, the more so as loss distributions are often far from being exactly known. One could also argue that the presented loss size distribution itself is unrealistic, that real distributions are quite smooth, far from having a discrete tail with such few values taken on. But it would be easy to construct a very smooth distribution being very close to the above, hence yielding very similar numerical results.

Anyway, we will see in the following that even very common loss models produce somewhat surprising effects.

2. Inflation leverage

We keep looking separately at the loss frequency and the loss severity of the layer. As the expected loss is the product of these one could think that it is sufficient to look just at this final figure. However, we want to take into account that in practice we do not have the models and model parameters ready – we have to infer them from data. In order to make the best out of the information given by the data it is
generally advisable to regard loss numbers and average losses separately, the more so as there is an important difference in what they can tell:

- The average layer loss can always be calculated from the reported data. If it changes over time (and this is not obscured by heavy random fluctuations) this should help detect inflation.
- The frequency of the layer losses may change due to inflation but it will change anyway if the ground-up loss frequency changes. If we are unable to quantify the latter (exactly) we cannot say (exactly) how much of the frequency change is due to inflation. In this case the empirical loss frequencies will not help much detect inflation.

**Definition:** We call

- frequency inflation: Increase of the layer loss frequency due to inflation
- severity inflation: Increase of the layer loss severity due to inflation
- total layer loss inflation: Increase of the expected layer loss due to inflation

The latter is the superposition of the two former. From the reinsurance example above we see that (at least theoretically) the “inflations” defined here can be extremely different from the ground-up inflation, they may be much larger and in the case of limited layers the severity inflation can have the opposite sign as the ground-up inflation.

In the following we look at marginal effects by assuming an (infinitesimally) small inflation rate, i.e. $g = 1 + h$ with a very small $h$.

The above inflation rates can be written as certain multiples of $h$ (at least for smooth distributions, see the details in the proposition below):

- frequency inflation $= L_{fr} h$
- severity inflation $= L_{sev} h$
- total layer loss inflation $= L_{tot} h$

**Definition:** We call

- $L_{fr}$: frequency inflation leverage
- $L_{sev}$: severity inflation leverage
- $L_{tot}$: total layer loss inflation leverage

Leverages greater than one mean the impact of inflation is more than proportionate. High leverages mean that the loss exposure of the excess insurance is very sensitive to ground-up inflation.

**Proposition 1:** For continuous $F$ the total layer loss inflation is well defined and equals

$$L_{tot} = 1 + \frac{d \cdot S(d) - (c + d)S(c + d)}{E(\min((X - d^+)\wedge c))} = 1 + \frac{d - (c + d)S(c + d|X > d)}{E(\min(X - d,c)|X > d)}$$

(2.1)

In case of infinite layers we have

$$L_{tot} = 1 + \frac{d \cdot S(d)}{E((X - d)^+)} = 1 + \frac{d}{E(X - d|X > d)}$$

(2.2)

If $F$ has a density $f$ then the frequency inflation leverage is well defined and equals

$$L_{fr} = \frac{d \cdot f(d)}{S(d)} = -d \cdot (\ln S)'(d)$$

(2.3)

In this case the severity leverage is well defined, too, and is equal to the difference between the total and the frequency leverage.

**Proof:** For each of the three formulae the second equivalence is clear.
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(2.3): The frequency at the attachment point after inflation equals
\[ E(N)P(X^* > d) = E(N)S(d/(1+h)), \] thus the frequency inflation is equal to \[ \frac{S(d/(1+h))}{S(d)} - 1. \]

If we develop this into a Taylor series around \( h \) and take the derivative at \( h = 0 \) of this expression we have
\[ \left( -f \left( \frac{d}{1+h} \right) \right) \left( -\frac{d}{(1+h)^2} \right). \]
evaluating at \( h = 0 \) we finally get \( \frac{d}{S(d)} \).

(2.1): The risk premium of the layer \( c \) \( x \) \( s \) \( d \) after inflation equals
\[ E(N)E(\min((X^* - d)^+, c)) = E(N) \int_{c}^{d} S^*(x) dx = E(N) \int_{c}^{d} S(x / (1+h)) dx, \] thus the total layer loss inflation is equal to \[ \frac{\int_{d}^{c+d} S(x / (1+h)) dx}{\int_{d}^{c+d} S(x) dx}. \]

To get again the first-order element of the Taylor series we rewrite the numerator as
\[ (1+h) \int_{d/(1+h)}^{c+d/(1+h)} S(u) du - \int_{d}^{c+d} S(x) dx = h \int_{d}^{c+d} S(x) dx + \int_{d/(1+h)}^{c+d/(1+h)} S(x) dx - \int_{c}^{d} S(x) dx = \]
\[ = h \int_{d/(1+h)}^{c+d/(1+h)} S(x) dx + \left( d - \frac{d}{1+h} \right) S(\xi_h) - \left( c + d - \frac{c + d}{1+h} \right) S(\xi_h), \]
where \( \xi_h \) is an appropriately chosen value between \( d \) and \( \frac{d}{1+h} \) according to the mean value theorem (recall \( S \) is continuous), and analogously \( \xi_h \). To take the derivative at \( h = 0 \) of this expression we have to divide by \( h \) and let \( h \) tend to zero, which yields \[ \int_{d}^{c+d} S(x) dx + d \cdot S(d) - (c + d) S(c + d). \]
Dividing by the nominator we get the final result: \( L_{\text{tot}} = 1 + \frac{d \cdot S(d) - (c + d) S(c + d)}{E(\min((X - d)^+, c))}. \)
The proof of (2.2) is analogous; here the upper bounds of the integrals are infinite, thus the negative term does not appear.

It remains to prove that \( L_{\text{tot}} = L_{\text{fr}} + L_{\text{sev}} \), which is obvious. Small inflation lets the total layer loss increase by the factor \( (1 + L_{\text{fr}}) \) \( (1 + L_{\text{sev}})h \), which in first order terms equals \( 1 + (L_{\text{fr}} + L_{\text{sev}})h \).

The formula for the frequency inflation leverage can be interpreted in two ways. Firstly it is the product of \( d \) with the hazard rate of the distribution \( F \). Secondly it equals the so-called local Pareto alpha (see Riegel), which is the slope of \( S \) on double-logarithmic paper (being constant in the Single-parameter Pareto case).
The formula for the total layer loss inflation leverage shows that inflation can be less than proportionate for finite layers, which occurs if \( xS(x) \) is an increasing function in the area of the layer.

Checking the proof we see that the conditions can be weakened: It is sufficient that \( F \) be continuous at \( d \) and \( c+d \). For the frequency inflation leverage we only need differentiability at \( d \). In particular the proposition can be applied to arbitrary distributions having a smooth tail for losses exceeding a threshold \( \theta \), provided that \( d > 0 \).
Notice that in such cases the inflation leverages for layers in the tail area depend on the parameters of the tail (conditional distribution) only. The distribution of the small losses does not matter at all.

In the following we will study two examples of such tails, namely the two cases of the Generalized Pareto Distribution (GPD) as defined in Extreme Value Theory (see Embrechts et al.) having infinite support on the positive axis: the Exponential distribution and the (proper) GPD.

3. Exponential tails

The Exponential distribution is popular in insurance to model losses having a not-too-heavy tail. We treat more generally distributions with Exponential tail, i.e. the conditional survival function beyond a threshold \( \theta \) is as follows:

\[
S(x|X > \theta) = \exp\left(-\frac{x-\theta}{\mu}\right).
\]

(Of course this includes the Exponential distribution itself as the special case \( \theta = 0 \).) Now we calculate the inflation leverages for layers in the tail area.

**Proposition 2**: If \( X \) has an Exponential tail beyond \( \theta \) then for layers \( c \) to \( d \) with \( d > \theta \) the inflation leverages are well defined and we have:

\[
L_{fr} = \frac{d}{\mu}, \quad L_{sev} = 1 - \frac{c/\mu \exp(-c/\mu)}{1 - \exp(-c/\mu)} \quad \text{(finite layer)}, \quad L_{sev} = 1 \quad \text{(infinite layer)}.
\]

**Proof**: We have a smooth tail and can apply Proposition 1. We will use the R.H.S.s of (2.1) and (2.3). As for the frequency, we notice that \( \ln(S(x)) = -x/\mu + \text{const.} \)

Taking the derivative at \( d \) and multiplying by \(-d\) we get \( L_{fr} \) as stated above.

The tail distribution in excess of \( d \), which is the “upper part” of the tail beyond \( \theta \), is as follows:

\[
S(x|X > d) = \exp\left(-\frac{x-d}{\mu}\right).
\]

From this we can calculate the expected layer loss as

\[
E(\min(X-d,c)|X > d) = \mu \left(1 - \exp\left(-\frac{c}{\mu}\right)\right).
\]

Plugging this into (2.1) we get \( L_{tot} = 1 + \frac{d - (c + d) \exp\left(-\frac{c}{\mu}\right)}{\mu \left(1 - \exp\left(-\frac{c}{\mu}\right)\right)} \), which after some algebra turns out to be equal to

\[
1 + \frac{d}{\mu} - \frac{c/\mu \exp(-c/\mu)}{1 - \exp(-c/\mu)}.
\]

Subtracting \( L_{fr} \) we get \( L_{sev} \) as stated above.

Taking \( c \) to infinity the last summand vanishes and we get the very simple formula for unlimited layers.

The frequency leverage is a function of the retention \( d \) and the Exponential parameter \( \mu \) only. More precisely it is a linear function of \( d \). The higher the layer, the higher the frequency inflation leverage. Conversely, the smaller \( \mu \) the higher the inflation leverage. If \( d \) is greater than the Exponential parameter \( \mu \) the impact of inflation to the layer loss frequency is more than proportionate.
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The severity leverage is a function of $\mu$ and $c$ but does not depend on $d$. This is due to the well-known characterising memoryless property of the Exponential distribution: The so-called excess loss variable $X-d \mid X>d$, which models the losses “to the layer”, has the survival function $\exp\left(-\frac{x}{\mu}\right)$, which is the Exponential distribution itself. This property leads to another way to see that the severity leverage of unlimited layers must equal one, i.e. that inflation has a proportionate impact to the severity:

Recall that $S^*(x) = S(x/g)$. Thus if $X$ has an Exponential tail, i.e. $S(x \mid X > \theta) = \exp\left(-\frac{x-\theta}{\mu}\right)$, then we see easily that after inflation for $X^*$ we get a tail distribution

$$S^* (x \mid X^* > g \theta) = \exp\left(-\frac{x-g\theta}{g\mu}\right)$$

to a somewhat different tail threshold. Both parameters $\mu$ and $\theta$ increase proportionately with the ground-up inflation, but only the latter is observable from layer data: If $d > g\theta$ we get

$$S^* (x \mid X^* > d) = \exp\left(-\frac{x-d}{g\mu}\right),$$

which is independent of the original threshold $g\theta$. The new excess loss variable $X^*-d \mid X^*>d$ is thus Exponentially distributed with parameter $g\mu$, which means that the average excess loss is $g$ times that before inflation.

Looking for a moment at finite layers, the formulae of Proposition 2 show that here the severity inflation less than proportionate, however, only a bit less if the layer liability $c$ is much larger than the Exponential parameter $\mu$.

Generally in the Exponential case, if the observed loss data is not too scarce, it should be no major problem to infer the inflation rate from the average losses observed in the layer: Apart from random fluctuations the empirical severities grow proportionately with the ground-up inflation.

However, this is specific for the Exponential case. In the following we will see totally different effects, making clear how much the distribution tail geometry matters for the inflation sensitivity.

4. Generalized Pareto tails

The GPD as defined in Extreme value theory is a distribution for losses greater than a threshold $\theta$, having the survival function

$$S(x \mid X > \theta) = \left(1 + \xi \frac{x-\theta}{\mu}\right)^{-\frac{1}{\xi}}, \quad x > \theta.$$

We do not regard the case of negative $\xi$, having finite support. The limiting case $\xi=0$ is the Exponential distribution treated in the preceding section. Let from now be $\xi > 0$. Often only this case is called GPD, thus the expression proper GPD. We use a different parametrisation (see Scollnik), replacing $\xi$ by the inverse $\alpha$ (being very popular in insurance) and introducing $\lambda = \mu / \xi - \theta$. Note that $\lambda$ may be (slightly) negative, values $\lambda > -\theta$ are admissible. Now the survival function reads

$$S(x \mid X > \theta) = \left(\frac{\theta + \lambda}{x + \lambda}\right)^{\alpha}.$$

Note that for positive $\lambda$ we can admit the case $\theta = 0$. This is a model for ground-up losses, namely the Two-parameter Pareto (American Pareto) model. The case $\lambda = 0$ instead is the classical Single-parameter Pareto (European Pareto) model for large losses. The advantage of this parametrisation is the following. If we in order to study layers $c \times s \times d$ in the tail area, i.e. $d > \theta$, calculate the tail hitting the layer, we get

$$S(x \mid X > d) = \left(\frac{d + \lambda}{x + \lambda}\right)^{\alpha}.$$
The parameters $\alpha$ and $\lambda$ have not changed. Both are thus invariant to upwards conditioning, i.e. to the modeling of higher tails (which is not true for the above parameter $\mu$ and some other parameter variants being used instead of $\lambda$). Notice that as in the Exponential case this tail does not depend on $\theta$, the upper tail “forgets” where the tail began.

Proposition 3: If $X$ has a (proper) GPD tail beyond $\theta$ then for layers $c \leq d$ with $d > 0$ the inflation leverages are well defined and we have:

$$L_{fr} = \frac{d}{d + \lambda} \alpha, \quad L_{lev} = \frac{\lambda}{d + \lambda} \alpha (\text{infinite layer, } \alpha > 1),$$

$$L_{lev} = \frac{\lambda}{d + \lambda} \left(1 - \frac{z - 1}{z H(z)}\right), \quad z = \frac{c + d + \lambda}{d + \lambda}, \quad H(z) = \frac{z^{\alpha-1} - 1}{\alpha - 1} \text{ if } \alpha \neq 1, \text{ else } H(z) = \ln(z) \text{ (finite layer).}$$

Proof: We have a smooth tail and can apply Proposition 1. We will use the R.H.S.s of (2.1) and 2.3).

As for the frequency, we notice that $\ln(S(x)) = -\alpha \ln(x + \lambda) + \text{const.}$

Taking the derivative at $d$ and multiplying by $-d$ we get $L_{fr}$ as stated above.

Notice that the expected layer loss $E(\min(X - d, c) | X > d) \text{ equals } \frac{d + \lambda}{\alpha - 1} \left(1 - \left(\frac{d + \lambda}{c + d + \lambda}\right)^{\alpha-1}\right)$ in case $\alpha \neq 1$ and $(d + \lambda) \ln\left(\frac{c + d + \lambda}{d + \lambda}\right)$ in case $\alpha = 1$.

By the definition of $H$ we can unite both cases to the expression $(d + \lambda) H(z) z^{1-\alpha}$.

Plugging this into (2.1) and noting that (for any $\alpha$) $\frac{z^\alpha - z}{z H(z)} = \alpha - 1$ we get

$$L_{tot} = 1 + \frac{d - (c + d) z^{-\alpha}}{(d + \lambda) H(z) z^{1-\alpha}} = 1 + \frac{d}{d + \lambda} \left(\frac{z - c + d}{z H(z)}\right) = 1 + \frac{d}{d + \lambda} \left(\frac{z^\alpha - z + z - c + d}{z H(z)}\right) =$$

$$= 1 + \frac{d}{d + \lambda} \left(\frac{z - c + d}{z H(z)}\right) = \frac{\lambda}{d + \lambda} \alpha + \frac{1}{d + \lambda} dz - (c + d).$$

As $dz - (c + d) = -\lambda (z - 1)$ we finally get $L_{tot} = \frac{d}{d + \lambda} \alpha + \frac{\lambda}{d + \lambda} \left(1 - \frac{z - 1}{z H(z)}\right)$.

Subtracting $L_{fr}$ we get $L_{lev}$ as stated above.

If in the case $\alpha > 1$ (where the expectation of unlimited layers exist) we take $c$ to infinity the last term vanishes and we get the very simple formula for unlimited layers.

The frequency leverage is a function of the retention $d$ and the parameters $\alpha$ and $\lambda$, being however much different from the Exponential case. For positive $\lambda$ it is an increasing function of $d$, for negative $\lambda$ a decreasing one, however, converging to $\alpha$ in both cases. For large $d$ the frequency leverage is almost independent of $d$. Generally it is a linear function of $\alpha$. The higher this parameter (i.e. the thinner the GPD tail) the larger the impact of inflation on the loss frequency – at least this result is analogous to the Exponential case.
The severity leverage is a function of \(c, d, \lambda, \) and \(\alpha\). It can be positive and negative according to the sign of \(\lambda\). (It is easy to see that \(1 - \frac{z-1}{zH(z)}\) is always between 0 and 1.) Hence, for negative \(\lambda\) inflation has a converse impact on the severity. If \(d\) is large compared to \(\lambda\) the severity leverage is almost linear in \(\lambda\) and generally very small. In this case inflation has a very low impact on the layer severity, which will make it difficult to infer inflation from changes in the empirical severity: Systematic changes will be hard to distinguish from random fluctuations.

For infinite layers \((\alpha > 1)\) the total layer loss inflation leverage \(L_{\text{tot}} = \frac{d}{d+\lambda} \alpha + \frac{\lambda}{d+\lambda}\) can be interpreted as the weighted mean of the parameter \(\alpha\) and the local Pareto alpha at the retention \(d\), being equal to the frequency inflation leverage \(\frac{d}{d+\lambda} \alpha\). In fact it is easy to see that here

\[
L_{\text{tot}} = \frac{1}{\alpha} + \left(\frac{d}{d+\lambda}\right) \alpha \left(1 - \frac{1}{\alpha}\right).
\]

Let us take a closer look at the survival function. Recall that \(S^*(x) = S(x/g)\). Thus if \(X\) has a GPD tail, i.e. \(S(x|X > \theta) = \left(\frac{\theta + \lambda}{x + \lambda}\right)^\alpha\), then we see easily that after inflation for \(X^*\) we get again a GPD tail \(S^*(x|X^* > g\theta) = \left(\frac{g\theta + g\lambda}{x + g\lambda}\right)^\alpha\) referring to a somewhat different tail threshold. Finally, if \(d > g\theta\) we get \(S^*(x|X^* > d) = \left(\frac{d + g\lambda}{x + g\lambda}\right)^\alpha\).

The tail after inflation is GPD as before with an unchanged parameter \(\alpha\) and parameters \(\theta\) and \(\lambda\) having changed proportionately with the ground-up inflation (like the Exponential parameters above). However, the last formula shows that again the original threshold \(g\theta\) is not observable if we only see the tail in excess of \(d\). In theory one could now try to infer inflation from changes in the parameter \(\lambda\), however, in practice this seems hopeless for large \(d\): The shape of the tail in excess of \(d\) depends on \(g\) only via the sum of \(g\lambda\) with the much larger \(d\).

We have not yet regarded the special case \(\lambda = 0\), being the worst case for the inference of the ground-up inflation from layer loss data.

**Corollary:** If \(X\) has a Single-Parameter Pareto tail beyond \(\theta\) then for layers \(c < x < d\) with \(d > \theta\) the inflation leverages are well defined and we have:

\[
L_{\text{fr}} = \alpha, \quad L_{\text{sev}} = 0, \quad L_{\text{tot}} = \alpha.
\]

Here the frequency leverage is a constant. (In fact the local Pareto alpha is a constant.) The severity leverage is zero, such that small inflation, and thus consequently also large inflation, does not change the severity at all. If we therefore observe the empirical average losses in a layer from year to year and detect changes, these be solely due to random fluctuation.

If in this case we want to infer inflation from data, we have to look at the layer loss frequency, which could be problematic as there might be interference with frequency changes for reasons other than inflation. However, if such other effects are quantifiable or absent the frequency inflation can detect inflation. See Brazauskas es al. showing how in this case the inflation rate can be estimated together with the Pareto \(\alpha\).
Now we calculate the exact (not the marginal) impact of inflation to a layer in the Single-parameter Pareto case. If the ground-up losses increase by a factor \( g \) and \( d \) is greater than the tail thresholds (before and after inflation) \( \theta \) and \( g \theta \) then the layer severity does not change. Thus the layer risk premium changes exactly as the layer loss frequency, increasing by the factor

\[
\frac{S(d/g)}{S(d)} = \frac{S(\theta)\left(\frac{\theta}{d/g}\right)^\alpha}{S(\theta)\left(\frac{\theta}{d}\right)^\alpha} = g^\alpha
\]

The property that European Pareto tails are inflation invariant, \( S^*(x|X > d) = \left(\frac{d}{x}\right)^\alpha = S(x|X > d) \), has by the way a counterpart in physics, where phenomena that look the same if all quantities are increased by a factor are called scale invariant. In fact the (Single-parameter) Pareto distribution appears in collections of such phenomena (albeit taking several different names).

For completeness we show that the property that the tail does not change with inflation is a characterization of the Single-parameter Pareto distribution. Let for all \( x \) greater than a certain threshold \( d \) be

\[
S^*(x|X > d) = S(x|X > d), \quad \text{i.e.} \quad \frac{S(x/g)}{S(d/g)} = \frac{S(x)}{S(d)}, \quad \text{i.e.} \quad \frac{S(x/g) - S(x)}{S(x)} = \frac{S(d/g) - S(d)}{S(d)}.
\]

The latter expression means that the frequency inflation does not depend on \( x \) for \( x \geq d \). Thus the marginal frequency inflation is a constant, too. From (2.3) we get \( (\ln S)'(x) = \frac{\text{const.}}{x} \), which means that \( S(x) \) is a power curve for \( x \geq d \), thus \( S \) has a Single-parameter Pareto tail.

Single-parameter Pareto is certainly an extreme case. If \( X \) does not exactly but only roughly follow this distribution one could hope that the severity leverage is different from zero, helping to detect inflation from the loss record of the layer. However, the above results for the GPD make clear that this in practice often will not work, as one typically has situations where the layer retention \( d \) is large with respect to the parameter \( \lambda \), such that inflation has an extremely low effect on the layer severity. And unlike European Pareto the GPD is not just another very special loss distribution – Extreme Value Theory (see Embrechts et al.) tells that the upper tails of many distributions look very much like the GPD, thus will yield results being numerically very close to those of Proposition 3.

We look at two such cases now, which shall conclude the paper.

### 5. Examples from reinsurance

#### Property NatCat layers

The risk that an insurer suffers accumulation losses in Property insurance due to natural disasters (Earthquake, flood, windstorm, etc.) has made NatCat XL reinsurance one of the most common products in the industry. This kind of business is observed to be very heavy-tailed, thus is traditionally perceived as a rather “dangerous” area. However, while it certainly bears some huge risks like extreme random fluctuation, inflation (and the uncertainty stemming from it) is a minor issue, for two reasons.

Firstly, the ground-up inflation can be (at least roughly) inferred from a kind of data being available in addition to layer loss data: the sums insured of the single risks. These are usually updated every year (or at least regularly) in order to cater for the increase of the replacement value of the insured property.
Inflation and Excess Insurance

due to inflation. Although this inflation might not be exactly the same as the inflation for large accumulation losses, it is a fair indication.

Secondly, the typically very heavy tails of accumulation loss size distributions themselves make inflation a minor problem. If we fit such tails by a GPD, the resulting parameter $\alpha$ will mostly be small, in many cases even somewhat lower than 1. That means that for rather high layers the total layer loss inflation leverage is less than 1, thus the overall impact of inflation to the layer is less than proportionate. In this situation the Cat-XL reinsurance premiums do not need higher increases than the primary insurance premiums.

Motor Third Party Liability XLs

The risk of huge MTPL losses is catered for by XL reinsurance, too. This kind of cover is perceived as very critical, although the distribution tails are by far not the heaviest ones observed in the reinsurance industry. The reasons are as follows:

Firstly, the ground-up inflation is very difficult to estimate. Knowledge about the overall inflation in MTPL does not help too much, as this is dominated by the repair cost of cars. Losses in the million Euro range are totally different, with car repair being a minor head of damage. The biggest part of such losses is typically the compensation to be paid to one or more seriously injured and disabled victims of the car accident, containing a big loss of earnings and an even bigger cost of care component. Such indemnifications rise due to developments in the health sector, changes in the social system, and other issues being far away from the overall MTPL inflation. Worse than being just different, the inflation of large MTPL losses seems to be considerably higher, the difference being sometimes called superimposed inflation. For a fair assessment of the latter one has to go into the details of large loss data (see Swiss Re).

Secondly, the typically moderate tails of MTPL loss size distributions themselves make inflation a major issue. GPD fits will mostly yield a rather large parameter $\alpha$. There are considerable differences from country to country, however, it is not uncommon to see values greater than 2. That means that for rather high layers the total layer loss inflation leverage exceeds 2, thus the inflation rate in the XL cover is more than twice the ground-up inflation. In other words: Layering more than doubles inflation.

Here is a numerical example, for the sake of simplicity the Single-parameter Pareto case, say $\alpha = 2.2$. Assume the ground-up losses involving severely injureds have a yearly superimposed inflation of 4% in addition to an overall MTPL inflation rate of 3%, which means a total rate of 7% per year ($g = 1.07$). As stated above the layer risk premium has to be multiplied by $g^\alpha$, which equals 1.16. Thus while the primary MTPL premiums should rise by 3% per year, high MTPL layers need a yearly premium adjustment of 16%.

6. Conclusion

The extreme tail-dependency of the impact of inflation to excess insurance changes the view of which kind of risk is more “dangerous” and which is less. While usually heavy-tailed risks are perceived as the most problematic ones due to their high random fluctuation and a considerable model and parameter risk, now inflation – plus the inherent uncertainty if it is not exactly quantifiable – directs the attention to other business: Risks having very heavy tails (approximately GPD with small $\alpha$) are not too much inflation-sensitive, while risks having less heavy tails (approximately GPD with high $\alpha$ or Exponential) are very much exposed to inflationary effects, the more so as inflation may generate a dependency among many seemingly uncorrelated risks.
References


