Inflation and Excess Insurance

Michael Fackler
freelance actuary
Munich (Germany)
Question

Inflation from the ground up

Can we reconstruct this?

Inflation to the insurance cover

Can we calculate this?
(re)insurance layer

Structure of cover: c xs d

Effects of (positive) inflation:

- rising loss burden
- rising loss frequency
- loss severity?
Systematic study of inflationary effects

The total layer inflation is the superposition of the following two effects:

• **Frequency inflation** (at attachment point)
• **Severity inflation**

The distinction makes sense as only the latter is always observable – the former may be obscured by frequency variations of other kind.
Leverage effects

Assume a small inflation rate $h$. We have a resulting

- frequency inflation: $L_{fr} \times h$
- severity inflation: $L_{sev} \times h$

We call

- $L_{fr}$: frequency inflation leverage
- $L_{sev}$: severity inflation leverage
- $L_{tot}$: total layer inflation leverage, $L_{tot} = L_{fr} + L_{sev}$
General formulae

Loss size distribution $F(x)$ with density $f(x)$ and survival function $S=1–F$. We have

$$L_{fr}(d) = d \frac{f(d)}{S(d)}$$

$$L_{tot}(c, d) = 1 + \frac{d - (c + d)S(c + d|X > d)}{E(min(X - d, c)|X > d)}$$
Example 1: Exponential tail

\[ S(x|X > \theta) = \exp\left( -\frac{x - \theta}{\mu} \right) \]

\[ L_{fr} = d/\mu \]

\[ L_{sev} = 1 - \frac{c/\mu}{\frac{e^{-c/\mu}}{1-e^{-c/\mu}}} \]
Example 2: Generalized Pareto tail

\[ S(x | X > \theta) = \left( \frac{\theta + \lambda}{x + \lambda} \right)^\alpha \]

\[ L_{fr} = \frac{d}{d + \lambda} \alpha \]

\[ L_{sev} = \frac{\lambda}{d + \lambda} \left[ 1 - \frac{(\alpha - 1)(z-1)}{z^\alpha - z} \right], \quad z = \frac{c + d + \lambda}{d + \lambda} \]
Example 2b: Special case Pareto

\[
S(x | X > \theta) = \left( \frac{\theta}{x} \right)^\alpha
\]

\[L_{fr} = \alpha\]

\[L_{sev} = 0\]

\[L_{tot} = \alpha\]
In the real world:

- Parameters are not exactly known.
- Distributions are somewhat uncertain.

Frequent situation:
For high losses the GPD approximates the tail quite well and the numerical results are close to the Pareto case.
In the Pareto world:

- Inflation does not affect the severity,
- ... thus is only observable from the frequency inflation,
- ... which may be difficult to estimate due to interference with other frequency variations.
- The total layer inflation leverage equals the Pareto alpha.

For GPD tails the situation is almost the same.
Industry examples

**Nat Cat** accumulation XLs:
very heavy tail, say GPD with $\alpha = 0.8$
- layer inflation lower than from the ground up

**MTPL** losses in the million Euro range
GPD tail with rather high $\alpha > 2$
- layer inflation more than twice as high as f.g.u.
- dramatic consequences as ground-up losses already rise much faster than consumer prices
The End

Thanks for joining this talk.

Feedback welcome.

michael_fackler@web.de