Provisions for Loss Adjustment Expenses

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Loss Adjustment Expenses: Allocated vs. Unallocated

Allocated Loss Adjustment Expenses

- Expenses for Medical Examinations
- Appraiser Expenses
- Lawyer Expenses
- etc.
Loss Adjustment Expenses: Allocated vs. Unallocated

Allocated Loss Adjustment Expenses
- Expenses for Medical Examinations
- Appraiser Expenses
- Lawyer Expenses
- etc.

Unallocated Loss Adjustment Expenses
- Expenses for IT-Systems
- Salary for Claims Department
- etc.
The New York Method

Assume

\[ LAE = \varepsilon(Paid + RBNS + IBNR), \quad \varepsilon > 0. \]
The New York Method

Assume

$$LAE = \varepsilon(Paid + RBNS + IBNR), \quad \varepsilon > 0.$$  

Then

$$LAE_{Used} = \varepsilon Paid$$

and

$$LAE_{Prov} = \varepsilon IBNR$$
The New York Method

Assume

\[ LAE = \varepsilon(Paid + RBNS + IBNR), \quad \varepsilon > 0. \]

Then

\[ LAE_{Used} = \varepsilon(Paid + \omega RBNS), \quad \omega \in [0, 1], \]

and

\[ LAE_{Prov} = \varepsilon(IBNR + (1 - \omega)RBNS), \]
The New York Method

Assume

\[ LAE = \varepsilon (Paid + RBNS + IBNR), \quad \varepsilon > 0. \]

Then

\[ LAE_{Used} = \varepsilon (Paid + \omega RBNS), \quad \omega \in [0, 1], \]

and

\[ LAE_{Prov} = \varepsilon (IBNR + (1 - \omega)RBNS), \]

where

\[ LAE = LAE_{Used} + LAE_{Prov}. \]
Calibration of the New York Method

Zero-run-off of the LAE provisions:

\[
\varepsilon (IBNR_1 + (1 - \omega)RBNS_1) \\
+ \varepsilon (IBNR_{new}^2 + RBNS_{new}^2 + \Delta Paid_{new}) \\
= \varepsilon (IBNR_2 + (1 - \omega)RBNS_2) + \Delta LAE_{Used}.
\]
Calibration of the New York Method

Zero-run-off of the LAE provisions:

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\varepsilon (IBNR_1 + (1 - \omega)RBNS_1) \\
+ \varepsilon (IBNR_{new}^2 + RBNS_{new}^2 + \Delta Paid_{new}) \\
= \varepsilon (IBNR_2 + (1 - \omega)RBNS_2) + \Delta LAE_{Used}.
\]

Reordering gives

\[
\frac{1}{\varepsilon} \Delta LAE_{Used} = (IBNR_1 + (1 - \omega)RBNS_1) \\
+ (IBNR_{new}^2 + RBNS_{new}^2 + \Delta Paid_{new}) \\
- (IBNR_2 + (1 - \omega)RBNS_2).
\]
Calibration of the New York Method

Zero-run-off of the LAE provisions:

\[
\begin{align*}
\varepsilon (IBNR_1 + (1 - \omega)RBNS_1) \\
+ \varepsilon (IBNR_{new}^2 + RBNS_{new}^2 + \Delta Paid_{new}) \\
= \varepsilon (IBNR_2 + (1 - \omega)RBNS_2) + \Delta LAE_{Used}.
\end{align*}
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Reordering gives

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+ (IBNR_{new}^2 + RBNS_{new}^2 + \Delta Paid_{new}) \\
- (IBNR_2 + (1 - \omega)RBNS_2).
\]

Definition of run-off result for the claims provisions

\[
RO_2 = (IBNR_1 + RBNS_1) - (IBNR_{old}^2 + RBNS_{old}^2) - \Delta Paid_{old}.
\]
Continuation of calibration of the New York Method

Combining the run-off result and the rearranged Zero-run-off formula gives

\[
\frac{1}{\hat{\varepsilon}} \Delta LAE_{Used} = RO_2 + \Delta Paid + \omega (RBNS_2 - RBNS_1)
\]

or equivalently

\[
\hat{\varepsilon} = \frac{\Delta LAE_{Used}}{RO_2 + \Delta Paid + \omega (RBNS_2 - RBNS_1)}.
\]
Continuation of calibration of the New York Method

Combining the run-off result and the rearranged Zero-run-off formula gives

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\frac{1}{\hat{\varepsilon}} \Delta LAE_{Used} = RO_2 + \Delta Paid + \omega (RBNS_2 - RBNS_1)
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or equivalently

\[
\hat{\varepsilon} = \frac{\Delta LAE_{Used}}{RO_2 + \Delta Paid + \omega (RBNS_2 - RBNS_1)}.
\]

An approximation could be

\[
\hat{\varepsilon} \approx \frac{\Delta LAE_{Used}}{\Delta Paid}.
\]
Continuation of calibration of the New York Method

Combining the run-off result and the rearranged Zero-run-off formula gives

\[ \frac{1}{\varepsilon} \Delta LAE_{Used} = RO_2 + \Delta Paid + \omega (RBNS_2 - RBNS_1) \]

or equivalently

\[ \hat{\varepsilon} = \frac{\Delta LAE_{Used}}{RO_2 + \Delta Paid + \omega (RBNS_2 - RBNS_1)} \].

An approximation could be

\[ \hat{\varepsilon} \approx \frac{\Delta LAE_{Used}}{\Delta Paid} \].

The LAE provisions is then given by the following formula

\[ LAE_{Prov} = \varepsilon (IBNR + (1 - \omega) RBNS) \].
Overdispersed Poisson Model

The vectors \((X_{ij}, Y_{ij}), 1 \leq i, j \leq n\), are mutually independent and

\[
EX_{ij} = T_i \beta_j \quad \text{and} \quad VX_{ij} = \phi EX_{ij}
\]
\[
EY_{ij} = T'_i \beta'_j \quad \text{and} \quad VY_{ij} = \phi'EY_{ij}
\]

where

\[
\sum_{j=1}^{n} \beta_j = \sum_{j=1}^{n} \beta'_j = 1
\]
Overdispersed Poisson Model

Chain Ladder:

\[ \hat{T}_i = \frac{\sum_{j=1}^{n+1-i} X_{ij}}{\sum_{j=1}^{n+1-i} \beta_j} \]
Overdispersed Poisson Model

Chain Ladder: \[ \hat{T_i} = \frac{\sum_{j=1}^{n+1-i} X_{ij}}{\sum_{j=1}^{n+1-i} \beta_j} \]

Bornhuetter-Fergusson: \[ \hat{T_i} = T_i \]
Overdispersed Poisson Model

Chain Ladder:  
\[ \hat{T}_i = \frac{\sum_{j=1}^{n+1-i} X_{ij}}{\sum_{j=1}^{n+1-i} \beta_j} \]

Bornhuetter-Ferguson:  
\[ \hat{T}_i = T_i \]

Benkthander:  
\[ \hat{T}_i = \sum_{j=1}^{n+1-i} X_{ij} + T_i \left( 1 - \sum_{j=1}^{n+1-i} \beta_j \right) . \]
Provisions for Loss Adjustment Expenses

Overdispersed Poisson Model

Estimation

Overdispersed Poisson Model

Chain Ladder:
\[ \hat{T}_i = \frac{\sum_{j=1}^{n+1-i} X_{ij}}{\sum_{j=1}^{n+1-i} \beta_j} \]

Bornhuetter-Fergusson:
\[ \hat{T}_i = T_i \]

Benkthander:
\[ \hat{T}_i = \sum_{j=1}^{n+1-i} X_{ij} + T_i \left( 1 - \sum_{j=1}^{n+1-i} \beta_j \right) \]

\[ \hat{\beta}_k = \frac{1}{\prod_{j=k}^{n-1} \hat{f}_j} \left( 1 - \frac{1}{\hat{f}_{k-1}} \right), \quad 1 \leq k \leq n, \]
The Main Idea

\[ \tilde{\beta}_j = (\omega \beta_j + (1 - \omega) \beta'_j) \varepsilon \quad , \quad \omega \in [0, 1], \quad \varepsilon > 0 \]
The Main Idea
Calibration

Let

$$\Delta LAE_{Used} = \sum_{i=1}^{n} \tilde{T}_i \tilde{\beta}_{n+1-i}$$,
Calibration

Let

$$\Delta LAE_{Used} = \sum_{i=1}^{n} \tilde{T}_i \tilde{\beta}_{n+1-i},$$

such that

$$\hat{\varepsilon} = \frac{\Delta LAE_{Used}}{\sum_{i=1}^{n} \tilde{T}_i (\omega \beta_{n+1-i} + (1 - \omega) \beta'_{n+1-i})}.$$
Calibration

Let

$$\Delta LAE_{Used} = \sum_{i=1}^{n} \tilde{T}_i \tilde{\beta}_{n+1-i},$$

such that

$$\hat{\varepsilon} = \frac{\Delta LAE_{Used}}{\sum_{i=1}^{n} \tilde{T}_i (\omega \beta_{n+1-i} + (1 - \omega) \beta'_{n+1-i})}.$$

LAE provision:

$$LAE^{(k)}_{Prov} = \sum_{j=n+2-k}^{n} \tilde{T}_k \tilde{\beta}_j \quad 2 \leq k \leq n.$$
Cash Flow

\[
CF_{LAE}^{(k)} = \sum_{i=k+1-n}^{n} \tilde{T}_i \tilde{\beta}_{k+1-i}, \quad k > n.
\]
Run-Off Result

LAE allocation:

$$\Delta LAE_{Used}^{(k)} = \frac{\tilde{T}_k \tilde{\beta}_{n+1-k}}{\sum_{i=1}^{n} \tilde{T}_i \tilde{\beta}_{n+1-i}} \Delta LAE_{Used}.\]
Run-Off Result

LAE allocation:

\[ \Delta LAE_{Used}^{(k)} = \frac{\tilde{T}_k \tilde{\beta}_{n+1-k} \Delta LAE_{Used}}{\sum_{i=1}^{n} \tilde{T}_i \tilde{\beta}_{n+1-i}}. \]

Run-Off Result:

\[ RO_{LAE}^{(k)} = LAE_{Prov}(Old) - \left( \Delta LAE_{Used}^{(k)} + \sum_{j=n+2-k}^{n} \tilde{T}_k \tilde{\beta}_j \right), \]

where \( 1 \leq k \leq n - 1 \),
Questions or comments?