The Impact of Inflation on Loss Reserves

Comparison of Results

René Stephan

ASTIN 2011

Madrid, June 19-22, 2010
Agenda

1. Introduction
2. Projection of Inflation
3. Projection of Claim Payments
4. Results
5. Appendix
Subject of Analysis and Motivation

‘In the short run inflation is an economic phenomenon. In the long run it is a political one.’ (Economist, June 5th 2010)

Subject of Analysis

- Measuring the impact of inflation on the undiscounted loss reserves.
- Analysis of German retail business, Fire and General Third Party Liability (GTPL).
- Assumption: Fire exposed to producer prices and GTPL exposed to salary inflation.
- No distinction between sub-lines or types of claims.

Motivation

- Qis 5 requires to identify the type of inflation particular cash-flows are exposed to and to apply appropriate assumptions for future inflation.
- To assess the risk due to inflation adequately,
  a. the high inflation rates in the 1970s and 1980s and
  b. the random nature of the run-off patterns have to be accounted for.
Approach

- Projection of cash-flows from individual LoBs and simultaneously from two LoBs.
- Implicite and explicite treatment of inflation.
- Results derived by simulation approach.
Inflation indices $l_t$ ($t$ = year) are in general non-stationary, i.e., first and second moments are not constant in time.

The rate of inflation is measured as logarithmic difference of order one, i.e.,

$$\Delta y_t = y_t - y_{t-1}$$

with $y_t = \ln(l_t)$. 
Time Series Models of the Rate of Inflation $\Delta y_t$

**Autoregressive Process of Order 1, AR(1),**

$$\Delta y_t = \nu + a\Delta y_{t-1} + u_t,$$

with $u_t$ white noise.

**Vector Autoregressive Process of Order 1, VAR(1),**

$$
\begin{pmatrix}
\Delta y_{1t} \\
\Delta y_{2t}
\end{pmatrix} = 
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix} + 
\begin{pmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{pmatrix}
\begin{pmatrix}
\Delta y_{1,t-1} \\
\Delta y_{2,t-1}
\end{pmatrix} + 
\begin{pmatrix}
u_1 \\
\nu_2
\end{pmatrix},
$$

with $u_t = (u_{1t}, u_{2t})'$ two-dimensional white noise.

**Vector Error Correction Model (VECM),**

$$
\begin{pmatrix}
\Delta y_{1t} \\
\Delta y_{2t}
\end{pmatrix} = 
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix} + 
\begin{pmatrix}
\alpha_1 & -\alpha_1 \beta_1 \\
\alpha_2 & -\alpha_2 \beta_1
\end{pmatrix}
\begin{pmatrix}
y_{1,t-1} \\
y_{2,t-1}
\end{pmatrix} + 
\begin{pmatrix}
\gamma_{11,1} & \gamma_{12,1} \\
\gamma_{21,2} & \gamma_{22,2}
\end{pmatrix}
\begin{pmatrix}
\Delta y_{1,t-1} \\
\Delta y_{2,t-1}
\end{pmatrix} + 
\begin{pmatrix}
u_1 \\
\nu_2
\end{pmatrix},
$$

with $u_t = (u_{1t}, u_{2t})'$ two-dimensional white noise.
Univariate Time Series Approach

Inflation $\Delta y_t$ as Autoregressive Process of Order 1, AR(1),

$$\Delta y_t = \nu + a \Delta y_{t-1} + u_t$$

with intercept $\nu$, parameter $a$ and $u_t$ normal white noise, ie $u_t \sim WN(0, \sigma^2)$.

Parameter Estimates, Time Period 1960 to 2010

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\nu}$</th>
<th>$\hat{a}$</th>
<th>$\hat{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Prices</td>
<td>0.0049</td>
<td>0.8100</td>
<td>0.0096</td>
</tr>
<tr>
<td>Producer Prices</td>
<td>0.0125</td>
<td>0.4271</td>
<td>0.0259</td>
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<tr>
<td>Labour Costs</td>
<td>0.0084</td>
<td>0.8309</td>
<td>0.0197</td>
</tr>
</tbody>
</table>

Example: Consumer Prices,

$$\Delta y_t = 0.0049 + 0.8100 \times \Delta y_{t-1} + u_t,$$

with $u_t \sim WN(0; 0.0096^2)$. 

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Multivariate Time Series Approach

Producer Prices and Salary Inflation as VAR(1),

\[
\begin{pmatrix}
\Delta y_{1t} \\
\Delta y_{2t}
\end{pmatrix} = \begin{pmatrix}
0.0069 \\
0.0086
\end{pmatrix} + \begin{pmatrix}
0.3620 & 0.1527 \\
-0.1788 & 0.8718
\end{pmatrix} \begin{pmatrix}
\Delta y_{1,t-1} \\
\Delta y_{2,t-1}
\end{pmatrix} + \begin{pmatrix}
u_{1t} \\
u_{2t}
\end{pmatrix}
\]

with

\[
E[u_t u'_t] = \begin{pmatrix}
8.015e-04 & 5.580e-05 \\
5.580e-05 & 3.677e-04
\end{pmatrix}.
\]

Producer Prices and Salary Inflation as VECM,

\[
\Delta y_{1t} = 0.371 - 0.159 (y_{1,t-1} - 0.498 y_{2,t-1}) + 0.365 \Delta y_{1,t-1} + 0.036 \Delta y_{2,t-1} + u_{1t},
\]

\[
\Delta y_{2t} = 0.095 - 0.038 (y_{1,t-1} - 0.498 y_{2,t-1}) - 0.178 \Delta y_{1,t-1} + 0.844 \Delta y_{2,t-1} + u_{2t},
\]

with

\[
E[u_t u'_t] = \begin{pmatrix}
5.531e-04 & 1.583e-04 \\
1.583e-04 & 3.583e-04
\end{pmatrix}.
\]

Parameter estimates are based on time period 1960 to 2010.
Possible realizations of the future rate of inflation (2011 - 2035),

Rate of Inflation as VAR(1) process

Rate of Inflation as VECM
Producer Prices: Residuals, VECM

Autocorrelation Function (ACF)

Standardized Residuals

Histogram: Standardized Residuals

QQ–Plot
Salaries Inflation: Residuals, VECM

Autocorrelation Function (ACF)

Standardized Residuals

Histogram: Standardized Residuals

QQ-Plot
Projection of Claim Payments

- Conversion of historical nominal payments to common price level, here: 2010.
  Assumption: Price level of incremental claim payment depends on year of payment, that is, on the transaction date.
- Projection of cumulative real claim payments $\tilde{C}_{i,k+1}$ of a single accident year
  - for individual LoB,
    $$\tilde{C}_{i,k+1} = \tilde{C}_{i,k} \hat{f}_{k+1} + \tilde{C}_{i,k} \hat{s}_{i,k+1} Z_{i,k+1},$$
    with $Z_{i,k+1} \sim N(0, 1)$, and
  - for two LoBs simultaneously,
    $$\begin{pmatrix} \tilde{C}_{i,k+1}^{(1)} \\ \tilde{C}_{i,k+1}^{(2)} \end{pmatrix} = \begin{pmatrix} \tilde{C}_{i,k}^{(1)} & 0 \\ 0 & \tilde{C}_{i,k}^{(2)} \end{pmatrix} \begin{pmatrix} \hat{f}_{k+1}^{(1)} \\ \hat{f}_{k+1}^{(2)} \end{pmatrix} + \tilde{\Sigma}_{i,k+1}^{1/2} \begin{pmatrix} Z_{i,k+1}^{(1)} \\ Z_{i,k+1}^{(2)} \end{pmatrix},$$
    with $Z_{i,k+1} = (Z_{i,k+1}^{(1)}, Z_{i,k+1}^{(2)})' \sim N_2(0, I)$. 

Development Factors Fire and GTPL

Development factors $\hat{f}_{k+1}$, $\hat{\sigma}_{k+1}$ and $\hat{\rho}_{k+1}$.
Method A1: LoBs individually, inflation implicitly, prediction error according to T. Mack

<table>
<thead>
<tr>
<th>Accident-year</th>
<th>Fire Loss O/S</th>
<th>Fire $MSEP^{1/2}$</th>
<th>Fire Loss O/S</th>
<th>GTPL $MSEP^{1/2}$</th>
<th>GTPL Loss O/S</th>
<th>GTPL $MSEP^{1/2}$</th>
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</thead>
<tbody>
<tr>
<td>2000</td>
<td>0</td>
<td>0</td>
<td>19</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>-0</td>
<td>0</td>
<td>27</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>1</td>
<td>2</td>
<td>31</td>
<td>13</td>
<td></td>
<td></td>
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<tr>
<td>2003</td>
<td>16</td>
<td>24</td>
<td>141</td>
<td>55</td>
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<tr>
<td>2004</td>
<td>-59</td>
<td>152</td>
<td>182</td>
<td>126</td>
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<tr>
<td>2005</td>
<td>-88</td>
<td>168</td>
<td>309</td>
<td>190</td>
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<tr>
<td>2006</td>
<td>-4</td>
<td>179</td>
<td>1,269</td>
<td>530</td>
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</tr>
<tr>
<td>2007</td>
<td>54</td>
<td>251</td>
<td>1,188</td>
<td>633</td>
<td></td>
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<tr>
<td>2008</td>
<td>371</td>
<td>548</td>
<td>1,868</td>
<td>1,535</td>
<td></td>
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<tr>
<td>2009</td>
<td>1,508</td>
<td>749</td>
<td>2,620</td>
<td>1,735</td>
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<tr>
<td>2010</td>
<td>8,217</td>
<td>2,928</td>
<td>3,142</td>
<td>4,007</td>
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</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10,016</strong></td>
<td><strong>3,202</strong></td>
<td><strong>10,797</strong></td>
<td><strong>5,027</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Results

Method A1: LoBs individually, inflation implicitly, prediction error according to T. Mack
Method B1: LoBs individually, inflation implicitly, simulation approach
Method C1: LoBs individually, inflation explicitly as AR(1) process, simulation approach

<table>
<thead>
<tr>
<th></th>
<th>Fire Loss O/S</th>
<th>Fire $MSEP^{1/2}$</th>
<th>GTPL Loss O/S</th>
<th>GTPL $MSEP^{1/2}$</th>
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<tr>
<td>A1</td>
<td>10,016</td>
<td>3,202</td>
<td>10,797</td>
<td>5,027</td>
</tr>
<tr>
<td>B1</td>
<td>10,008</td>
<td>3,202</td>
<td>10.783</td>
<td>5,023</td>
</tr>
<tr>
<td></td>
<td>(-0.1%)</td>
<td>(0.0%)</td>
<td>(-0.1%)</td>
<td>(-0.1%)</td>
</tr>
<tr>
<td>C1</td>
<td>9,736</td>
<td>3,440</td>
<td>11,495</td>
<td>5,516</td>
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<tr>
<td></td>
<td>(-2.8%)</td>
<td>(7.4%)</td>
<td>(6.5%)</td>
<td>(9.7%)</td>
</tr>
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</table>

⇒ Implicit treatment of inflation leads to playing down the reserve volatility for Fire and GTPL.

⇒ The situation concerning reserve adequacy varies from LoB to LoB.
Results

Method A2: LoBs simultaneously, inflation implicitly, prediction error acc. to C. Braun
Method B2: LoBs simultaneously, inflation implicitly, simulation approach
Method C2: LoBs simultaneously, inflation explicitly as VAR(1) process, sim. approach
Method D2: LoBs simultaneously, inflation explicitly as VECM, simulation approach

<table>
<thead>
<tr>
<th></th>
<th>Fire Loss O/S</th>
<th>Fire $\text{MSEP}^{1/2}$</th>
<th>GTPL Loss O/S</th>
<th>GTPL $\text{MSEP}^{1/2}$</th>
<th>Total Loss O/S</th>
<th>Total $\text{MSEP}^{1/2}$</th>
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<tbody>
<tr>
<td>A2</td>
<td>10,016</td>
<td>3,202</td>
<td>10,797</td>
<td>5,027</td>
<td>20,813</td>
<td>6,762</td>
</tr>
<tr>
<td>B2</td>
<td>10,020</td>
<td>3,204</td>
<td>10,808</td>
<td>5,031</td>
<td>20,827</td>
<td>6,765</td>
</tr>
<tr>
<td>C2</td>
<td>9,698</td>
<td>3,434</td>
<td>11,374</td>
<td>5,449</td>
<td>21,072</td>
<td>7,274</td>
</tr>
<tr>
<td></td>
<td>(-3.2%)</td>
<td>(7.2%)</td>
<td>(5.3%)</td>
<td>(8.4%)</td>
<td>(1.2%)</td>
<td>(7.6%)</td>
</tr>
<tr>
<td>D2</td>
<td>9,563</td>
<td>3,372</td>
<td>11,303</td>
<td>5,404</td>
<td>20,866</td>
<td>7,199</td>
</tr>
<tr>
<td></td>
<td>(-4.5%)</td>
<td>(5.3%)</td>
<td>(4.7%)</td>
<td>(7.5%)</td>
<td>(0.3%)</td>
<td>(6.5%)</td>
</tr>
</tbody>
</table>

$\Rightarrow$ Results reassure that the implicit treatment of inflation underestimates reserve volatility.
Results

Method A2: LoBs simultaneously, inflation implicitly, prediction error acc. to C. Braun
Method C2: LoBs simultaneously, inflation explicitly as VAR(1) process, sim. approach
Method D2: LoBs simultaneously, inflation explicitly as VECM, simulation approach

<table>
<thead>
<tr>
<th></th>
<th>Fire</th>
<th>KH</th>
<th>Gesamt</th>
<th>Implicit Correl.</th>
<th>Total Correl.=0</th>
<th>Total Correl.=1</th>
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<tr>
<td>MSE $^{1/2}$</td>
<td>MSE $^{1/2}$</td>
<td>MSE $^{1/2}$</td>
<td>MSE $^{1/2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>3,202</td>
<td>5,027</td>
<td>6,762</td>
<td>0.32</td>
<td>5,960</td>
<td>8,229</td>
</tr>
<tr>
<td></td>
<td>(7.2%)</td>
<td>(8.4%)</td>
<td>(7.6%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>3,434</td>
<td>5,449</td>
<td>7,274</td>
<td>0.31</td>
<td>6,441</td>
<td>8,883</td>
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<tr>
<td></td>
<td>(5.3%)</td>
<td>(7.5%)</td>
<td>(6.5%)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td>3,372</td>
<td>5,404</td>
<td>7,199</td>
<td>0.31</td>
<td>6,370</td>
<td>8,776</td>
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</table>

⇒ The impact of inflation on the correlation between LoBs that are exposed to different rates of inflation (and that have different tail character) is at best loosely.
Producer Prices as AR(1): Residuals

Autocorrelation Function (ACF)

Standardized Residuals

Histogram: Standardized Residuals

QQ–Plot
Salary Inflation as AR(1): Residuals

Autocorrelation Function (ACF)

Standardized Residuals

Histogram: Standardized Residuals

QQ–Plot

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The Impact of Inflation on Loss Reserves

Madrid, June 19-22, 2010
Nominal Cumulative Claims Paid

**Fire**

<table>
<thead>
<tr>
<th>AY/DP</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
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<td>7,051</td>
<td>9,060</td>
<td>9,470</td>
<td>9,482</td>
<td>9,450</td>
<td>9,450</td>
<td>9,451</td>
<td>9,450</td>
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<td>9,656</td>
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<td>13,467</td>
<td>13,528</td>
<td>13,621</td>
<td>13,641</td>
<td>13,629</td>
<td>13,350</td>
<td>13,388</td>
<td>13,386</td>
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<td>8,542</td>
<td>8,465</td>
<td>8,484</td>
<td>8,481</td>
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<td>8,481</td>
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<td>13,220</td>
<td>13,225</td>
<td>13,225</td>
<td>13,052</td>
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<td>13,861</td>
<td>13,861</td>
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<td>17,176</td>
<td>18,285</td>
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<td>2008</td>
<td>10,760</td>
<td>22,253</td>
<td>24,166</td>
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<tr>
<td>2010</td>
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**GTPL**

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<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<th>11</th>
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<tr>
<td>1999</td>
<td>32</td>
<td>232</td>
<td>487</td>
<td>540</td>
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<td>1,453</td>
<td>1,464</td>
<td>1,485</td>
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<tr>
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<td>813</td>
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<td>1,277</td>
<td>1,304</td>
<td>1,307</td>
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<td>477</td>
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